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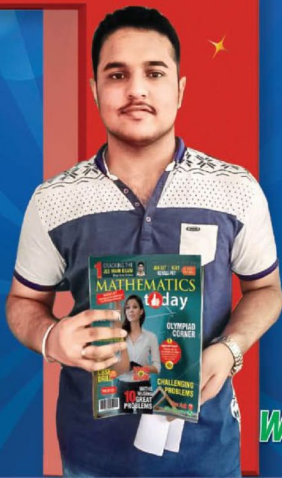
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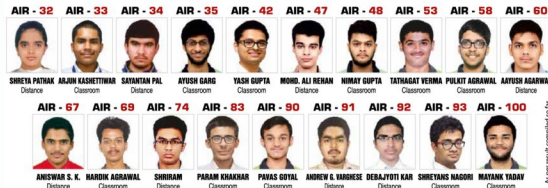
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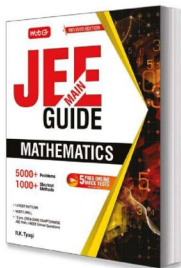
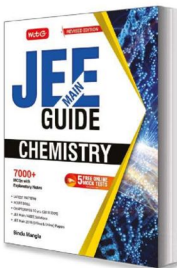
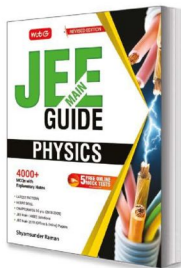
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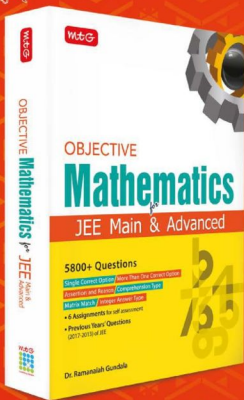


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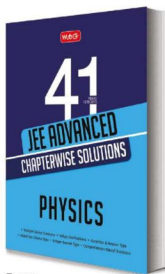
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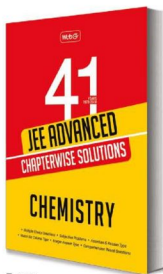
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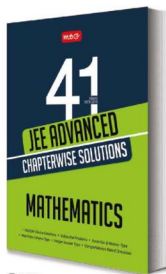
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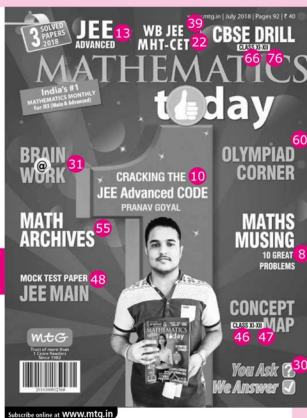
CONTENTS

Competition Edge

Class XI

Class XII

- 8 Maths Musing Problem Set - 187
- 10 Cracking the JEE Advanced Code
- 13 Solved Paper - JEE Advanced 2018
- 22 Solved Paper - MHT-CET 2018
- 28 Maths Musing Solutions
- 30 You Ask We Answer
- 31 Brain @ Work
- 39 Solved Paper - WB JEE 2018
- 48 Mock Test Paper JEE Main 2019 (Series 2)
- 55 Math Archives
- 57 Challenging Problems
- 60 Olympiad Corner
- 46 Concept Map
- 66 CBSE Drill (Series 3)
- 74 MPP-3
- 47 Concept Map
- 76 CBSE Drill (Series 3)
- 84 MPP-3



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JEE 13 ADVANCED **WB JEE 39 MHT-CET 22** **CBSE DRILL 66 76**

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BRAIN @ WORK 31

CRACKING THE JEE Advanced CODE 10 PRANAV GOYAL

MATH ARCHIVES 55

MOCK TEST PAPER 48 JEE MAIN

OLYMPIAD CORNER 60

MATHS MUSING 8 10 GREAT PROBLEMS

CONCEPT MAP 46 47

You Ask We Answer 30

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Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefiting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 187

JEE MAIN

- The equation $\text{Im} \left(\frac{iz-2}{z-i} \right) + 1 = 0, z \in \mathbb{C}, z \neq i$ represents a part of a circle having radius equal to
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 2
- In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radical is
(a) 715 (b) 725 (c) 730 (d) 745
- A triangle is formed with the vertices of n sided regular polygon. Then, the probability that the triangle has exactly two sides common with the side of the polygon is
(a) $\frac{3}{(n-1)(n-2)}$ (b) $\frac{6(n-4)}{(n-1)(n-2)}$
(c) $\frac{6(n-3)}{(n-1)(n-2)}$ (d) $\frac{6}{(n-1)(n-2)}$
- Let $A = \frac{2}{\sqrt{3}}e^{i\pi/2}, B = \frac{2}{\sqrt{3}}e^{-i\pi/6}, C = \frac{2}{\sqrt{3}}e^{-i5\pi/6}$, (where $i = \sqrt{-1}$) be three points forming a triangle ABC in the argand plane. Then $\triangle ABC$ is
(a) equilateral (b) isosceles
(c) scalene (d) none of these
- The coefficient of x^{203} in the expansion of $(x-1)(x^2-2)(x^3-3)\dots(x^{20}-20)$ is:
(a) -35 (b) 21 (c) 13 (d) 15

JEE ADVANCED

- If three unequal numbers p, q, r are in H.P and their squares are in A.P., then the ratio $p : q : r$ is:
(a) $1 - \sqrt{3} : -2 : 1 + \sqrt{3}$ (b) $1 : \sqrt{2} : -\sqrt{3}$
(c) $1 : -\sqrt{2} : \sqrt{3}$ (d) $1 + \sqrt{3} : -2 : 1 - \sqrt{3}$

COMPREHENSION

In a triangle ABC , D and E are points on BC and AC respectively, such that $2BD = DC$ and $3AE = 2EC$.

The lines AD and BE meet at P . The lines CP and AB meet at F .

- $AP : PD =$
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 2 : 3
- $BP : PE =$
(a) 3 : 4 (b) 4 : 3 (c) 5 : 4 (d) 4 : 5

INTEGER MATCH

- Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x \rightarrow -1} \frac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$.
If the maximum value of $f(x)$ is $\frac{9}{4}\lambda$, then λ is

MATRIX MATCH

- Match the following.

	List-I	List-II
P.	The probability that length of a randomly selected chord of a circle lies between $\frac{1}{4}$ and $\frac{3}{4}$ of its diameter, is	1. $1/30$
Q.	Three numbers are selected from the set $\{1, 2, 3, \dots, 10\}$. The probability that they form a G.P., is	2. $1/2$
R.	If x and y are selected from the set $\{1, 2, 3, \dots, 10\}$, the probability, that $x^3 + y^3$ is a multiple of 3, is	3. $1/4$
S.	A card is lost from a pack of 52 playing cards. If a card is drawn from the remaining cards, the probability that it is a spade, is	4. $1/3$

P	Q	R	S
(a) 2	1	4	3
(b) 1	2	3	4
(c) 3	4	1	2
(d) 4	3	2	1

KNOWLEDGE SERIES

(for JEE / Olympiad Aspirants)

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Knowledge Quiz - 8

Find all primes of the form $n^n + 1$,
which are less than 10^{19} .

Please send your detailed solution before 15th July 2018 to quiz@kcseducate.in along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 7

- Nishant Giri, (Class - XI), Delhi Public School, Dighoi, Assam.
- Arvind Krishna, (Class - XII), Sri Aurobindo Pre-university College, Shimoga, Karnataka.
- Viplav Patel, (Class - XII), St Basil's School, Basti, Uttar Pradesh.

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Cracking the JEE Advanced CODE



Pranav Goyal

- **MTG : Why did you appear for Engineering Entrance?**

Pranav : I was always interested in pursuing engineering due to my interest in Physics and Mathematics. I had made up my mind in class 9th itself that I would pursue engineering. I would like to take up Computer Science in IIT because of my interest in programming.

- **MTG : What exams you have appeared for and what is your rank in these exams?**

Pranav : I have appeared for the following exams :

JEE Advanced(2018)- 337/360 AIR-1, JEE Main(2018)-350/360 AIR-4, CBSE Board- 97.2% (Tricity Topper), KVPY Scholar, NTSE Scholar, INMO, INCHO, INPHO (AIR-5), INAO.

- **MTG : How many hours in a day did you study to prepare for the examination?**

Pranav : I used to study for 11-12 hours a day.

- **MTG : How much time does one require for serious preparation for this exam?**

Pranav : It varies from an individual to the other , because it actually depends upon the hardwork of an aspirant.

- **MTG : Any extra coaching?**

Pranav : Yes, from Sri Chaitanya Institute, Chandigarh. A coaching institute is extremely helpful for any JEE aspirant. It provides the correct study atmosphere for the preparation. Our institute laid to focus on our recreation activities also. Test series,

online tests and timed speed tests are very important.

- **MTG : Which subject/topic you were strong/weak at?**

Pranav : The primary challenge during the preparation was Chemistry. I had severe trouble in attempting to memorize both Organic and Inorganic Chemistry. Also, I possessed a very weak memory, hence, I would often forget the important stuff. Also, I faced problem with my accuracy. I found Physics , easy as the questions were simple, numericals based and previously studied. Maths was toughest as it required excellent solving skills and there were unexpected tricks.

- **MTG : Which books/ magazines you read?**

Pranav : A few books such as H.C. Verma for Physics, Pradeep for Chemistry and Physics theory. Solomons & Fryhle for Organic Chemistry and S.L. Loney for Mathematics. These majorly served as references. However, I studied my teachers' class notes completely and thoroughly.

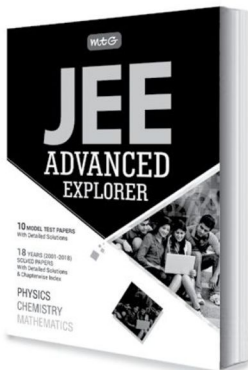
- **MTG : In your words what are the components of an ideal preparation plan?**

Pranav : Well, during preparation, I did expect to get a good rank in JEE Advanced. But never in my dreams did I imagine that I would be one of the toppers of JEE Advanced. In JEE advanced, I focussed more on conceptual

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understanding than rote memorizing which is essential for JEE Main. Also, I did skip some topics for JEE Advanced which were not included in its official syllabus.

Well, my preparation strategy was quite different after JEE Main. My study hours increased and I preferred to give speed tests to improve on my speed. Also, I avoided too much extra knowledge and focussed only on the part in JEE syllabus. I extensively used mock tests and sample papers. I gave all these in the form of timed tests. Any JEE aspirant has to solve atleast 8-9 previous years' papers. They are very important to give you a proper idea about difficulty level and a rough estimate regarding your subject time distribution. Also, these papers give you a fair idea of the topic that you need to practice and revise. I would like to suggest the same to all the aspirants.

- **MTG : What role did the following play in your success?**

- (a) Parents (b) Teachers
- (c) School

Pranav : (a) Parents: My parents, sacrificed a lot for my studies. My mother often had to stay awake late at night and wake up very early every morning. She did this only so that I could study. I owe my entire success to her.

(b) Teachers: Well, definitely my amazing teachers' stupendous guidance help me a lot. Beyond preparation teachers played major role in motivating me and helping me.

(c) School: I did my schooling from Bhavan Vidyalaya, Panchkula. The teachers of my school always motivated me and encouraged me to reach my goal.

- **MTG : Your family background?**

Pranav : My family consists of my grandparents, parents and younger brother. My father, Mr. Pankaj Goyal is a businessman. He works as a director of a pharma company. My mother, Mrs. Mamta Goyal is also a businesswoman like my dad.

- **MTG : What mistakes you think you shouldn't have made?**

Pranav : In the initial months, I used to struggle like any other child. Initially, I found it hard to cope with Mathematics. I really felt that, though I used to study hard but still at the beginning I didn't put all those efforts which were actually required.

- **MTG : How have MTG Magazines helped you in your preparation?**

Pranav : MTG Magazines really helped me as from these magazines I used to read the content thoroughly and practiced all the MCQs based on chapters.

- **MTG : Considering other hot career today, e.g., Biotechnology, etc., why did you still want to be an engineer?**

Pranav : I was always interested in taking up Engineering because of my interest in both Physics and Maths. I decided that I would take up Engineering in class 9th.

- **MTG : Was this your first attempt?**

Pranav : Yes, it was my first attempt.

- **MTG : What do you think is the secret of your success?**

Pranav : Secret of my success is that you should be focussed well, your study hours won't really matter. So, just stay focussed and calm. Have faith in your teachers and their guidance, and always ensure that your knowledge is conceptually sound.

- **MTG : What advice would you like to give our readers who are JEE aspirants?**

Pranav : Well, I would say,

"if you focus well, your study hours won't matter. However, if you don't focus, then too your study hours won't matter." Believe in your teachers and their guidance. Ensure that your knowledge is conceptually sound and do solve previous years' papers.

"Secret of my success is that you should be focussed well, your study hours won't really matter. So, just stay focussed and calm. Have faith in your teachers and their guidance, and always ensure that your knowledge is conceptually sound."

JEE ADVANCED

**SOLVED
PAPER**



PAPER-1

* ALOK KUMAR, B.Tech, IIT Kanpur

SECTION 1 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -2 In all other cases.

- For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

- For a non-zero complex number z , let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) FALSE?

- $\arg(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$
- The function $f: \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

- For any two non-zero complex numbers z_1 and z_2 , $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π .

- For any three given distinct complex numbers z_1, z_2 and z_3 , the locus of the point z satisfying the condition $\arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$, lies on a straight line.

- In a triangle PQR , let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?

- $\angle QPR = 45^\circ$
- The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$
- The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$
- The area of the circumcircle of the triangle PQR is 100π

- Let $P_1: 2x + y - z = 3$ and $P_2: x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?

- The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
- The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
- The acute angle between P_1 and P_2 is 60°
- If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2 , then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

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4. For every twice differentiable function $f: R \rightarrow [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

- (a) There exist $r, s \in R$, where $r < s$, such that f is one-one on the open interval (r, s)
 (b) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$
 (c) $\lim_{x \rightarrow \infty} f(x) = 1$
 (d) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

5. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two non-constant differentiable functions. If $f'(x) = (e^{f(x)} - g'(x))g'(x)$ for all $x \in R$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?

- (a) $f(2) < 1 - \log_e 2$ (b) $f(2) > 1 - \log_e 2$
 (c) $g(1) > 1 - \log_e 2$ (d) $g(1) < 1 - \log_e 2$

6. Let $f: [0, \infty) \rightarrow R$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{-t} f(t) dt$ for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE?

- (a) The curve $y = f(x)$ passes through the point $(1, 2)$
 (b) The curve $y = f(x)$ passes through the point $(2, -1)$
 (c) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$
 (d) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

SECTION 2 (MAXIMUM MARKS : 24)

- This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

7. The value of $((\log_2 9)^2)^{\log_2(\log_2 9)} \times (\sqrt{7})^{\log_4 7}$ is _____.

8. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is _____.

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is _____.

10. The number of real solutions of the equation

$$\sin^{-1} \left(\sum_{j=1}^{\infty} x^{j+1} - x \sum_{j=1}^{\infty} \left(\frac{x}{2} \right)^j \right) = \frac{\pi}{2} - \cos^{-1} \left(\sum_{j=1}^{\infty} \left(-\frac{x}{2} \right)^j - \sum_{j=1}^{\infty} (-x)^j \right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2} \right)$ is _____.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ and $[0, \pi]$, respectively.)

11. For each positive integer n , let

$$y_n = \frac{1}{n} ((n+1)(n+2) \dots (n+n))^n$$

For $x \in R$, let $[x]$ be the greatest integer less than or equal to x . If $\lim_{n \rightarrow \infty} y_n = L$, then the value of $[L]$ is _____.

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in R$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8 \cos^2 \alpha$ is _____.

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a \cos x + 2b \sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____.

14. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0)$, $Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ ($n > 1$). If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR , then the value of n is _____.

SECTION 3 (MAXIMUM MARKS : 12)

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
 - Each question has FOUR options. ONLY ONE of these four options corresponds to the correct answer.
 - For each question, choose the option corresponding to the correct answer.
 - Answer to each question will be evaluated according to the following marking scheme:
- Full Marks : +3 If ONLY the correct option is chosen.

- Zero Marks :** 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : -1 In all other cases.

Paragraph-I

Let S be the circle in the xy -plane defined by the equation $x^2 + y^2 = 4$.

(There are two questions based on PARAGRAPH-I, the questions (15-16) given below is one of them)

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x -axis and the y -axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1 . Let the tangents to S at E_1 and E_2 meet at E_3 , the tangents to S at F_1 and F_2 meet at F_3 , and the tangents to S at G_1 and G_2 meet at G_3 . Then, the points E_3, F_3 and G_3 lie on the curve

- (a) $x + y = 4$ (b) $(x - 4)^2 + (y - 4)^2 = 16$
 (c) $(x - 4)(y - 4) = 4$ (d) $xy = 4$

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N . Then, the mid-point of the line segment MN must lie on the curve

- (a) $(x + y)^2 = 3xy$ (b) $x^{2/3} + y^{2/3} = 2^{4/3}$
 (c) $x^2 + y^2 = 2xy$ (d) $x^2 + y^2 = x^2y^2$

Paragraph-II

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH-II the questions (17-18) given below is one of them)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 , and NONE of the remaining students gets the seat previously allotted to him/her is

- (a) $\frac{3}{40}$ (b) $\frac{1}{8}$ (c) $\frac{7}{40}$ (d) $\frac{1}{5}$

18. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

- (a) $\frac{1}{15}$ (b) $\frac{1}{10}$
 (c) $\frac{7}{60}$ (d) $\frac{1}{5}$

PAPER-2

SECTION 1 (MAXIMUM MARKS : 24)

- This section contains SIX (06) questions.
- Each question has FOUR options for correct answer(s). ONE OR MORE THAN ONE of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -2 In all other cases.

- For Example: If first, third and fourth are the ONLY three correct options for a question with second option being

an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

1. For any positive integer n , define $f_n : (0, \infty) \rightarrow \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^n \tan^{-1} \left(\frac{1}{1+(x+j)(x+j-1)} \right)$$

for all $x \in (0, \infty)$

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$). Then, which of the following statement(s) is (are) TRUE?

- (a) $\sum_{j=1}^5 \tan^2(f_j(0)) = 55$
 (b) $\sum_{j=1}^{10} (1 + (f_j'(0)) \sec^2(f_j(0))) = 10$

- (c) For any fixed positive integer n , $\lim_{x \rightarrow \infty} \tan(f_n(x)) = \frac{1}{n}$
 (d) For any fixed positive integer n , $\lim_{x \rightarrow \infty} \sec^2(f_n(x)) = 1$

2. Let T be the line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q , and also such that S_1 and S_2 touch each other at a point, say, M . Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1 . Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be F_2 . Let E_2 be the set of the mid-points of the line segments in the set F_2 . Then, which of the following statement(s) is (are) TRUE?

- (a) The point $(-2, 7)$ lies in E_1
 (b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2
 (c) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2
 (d) The point $\left(0, \frac{3}{2}\right)$ does NOT lie in E_1

3. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$\begin{aligned} -x + 2y + 5z &= b_1 \\ 2x - 4y + 3z &= b_2 \\ x - 2y + 2z &= b_3 \end{aligned}$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one

solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

- (a) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$
 (b) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
 (c) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
 (d) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q . Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is OQ . If the length of the minor

axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

- (a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1
 (b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$
 (c) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$
 (d) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

5. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ ($x, y \in \mathbb{R}, i = \sqrt{-1}$) of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

- (a) If L has exactly one element, then $|s| \neq |t|$
 (b) If $|s| = |t|$, then L has infinitely many elements
 (c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2
 (d) If L has more than one element, then L has infinitely many elements

6. Let $f: (0, \pi) \rightarrow \mathbb{R}$ be a twice differentiable function such that $\lim_{t \rightarrow x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$ for all $x \in (0, \pi)$.

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE?

- (a) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$
 (b) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$
 (c) There exists $\alpha \in (0, \pi)$ such that $f''(\alpha) = 0$
 (d) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION 2 (Maximum Marks : 24)

- This section contains EIGHT (08) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, -0.33, -30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

7. The value of the integral $\int_0^{1/2} \frac{1+\sqrt{x}}{((x+1)^2(1-x)^6)^{1/4}} dx$ is _____.

8. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is _____.

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X , then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____.

10. Let $f: R \rightarrow R$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2+5y)(5y-2),$$

then the value of $\lim_{x \rightarrow -\infty} f(x)$ is _____.

11. Let $f: R \rightarrow R$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in R$. Then, the value of $\log_e(f(4))$ is _____.

12. Let P be a point in the first octant, whose image Q in the plane $x+y=3$ (that is, the line segment PQ is perpendicular to the plane $x+y=3$ and the mid-point of PQ lies in the plane $x+y=3$) lies on the z -axis. Let the distance of P from the x -axis be 5. If R is the image of P in the xy -plane, then the length of PR is _____.

13. Consider the cube in the first octant with sides OP , OQ and OR of length 1, along the x -axis, y -axis and z -axis, respectively, where $O(0,0,0)$ is the origin.

Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT . If $\vec{p} = \vec{SP}$, $\vec{q} = \vec{SQ}$, $\vec{r} = \vec{SR}$ and $\vec{t} = \vec{ST}$, then the value of $|\vec{p} \times \vec{q}| \times (\vec{r} \times \vec{t})$ is _____.

14. Let $X = {}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \dots + 10({}^{10}C_{10})^2$, where ${}^{10}C_r, r \in \{1, 2, \dots, 10\}$ denote binomial coefficients.

Then, the value of $\frac{1}{1430} X$ is _____.

SECTION 3 (MAXIMUM MARKS : 12)

- This section contains FOUR (04) questions.
- Each question has TWO (02) matching lists: LIST-I and LIST-II.

• FOUR options are given representing matching of elements from LIST-I and LIST-II. ONLY ONE of these four options corresponds to a correct matching.

• For each question, choose the option corresponding to the correct matching.

• For each question, marks will be awarded according to the following marking scheme:

Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -1 In all other cases.

15. Let $E_1 = \left\{ x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0 \right\}$ and

$$E_2 = \left\{ x \in E_1 : \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right) \text{ is a real number} \right\}.$$

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f: E_1 \rightarrow R$ be the function defined by

$$f(x) = \log_e \left(\frac{x}{x-1} \right)$$

and $g: E_2 \rightarrow R$ be the function defined by

$$g(x) = \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right).$$

List-I	List-II
P. The range of f is	1. $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q. The range of g contains	2. $(0, 1)$
R. The domain of f contains	3. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
S. The domain of g is	4. $(-\infty, 0) \cup (1, \infty)$
	5. $\left(-\infty, \frac{e}{e-1}\right]$
	6. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

- (a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$
 (b) $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$
 (c) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$
 (d) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$

16. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5 .

- (i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.
 (ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.
 (iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.
 (iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

List-I		List-II	
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is :

- (a) $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 2; S \rightarrow 1$
 (b) $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 3$
 (c) $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 5; S \rightarrow 2$
 (d) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$

17. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola

in the xy -plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N . Let the area of the triangle LMN be $4\sqrt{3}$.

List-I		List-II	
P.	The length of the conjugate axis of H is	1.	8
Q.	The eccentricity of H is	2.	$\frac{4}{\sqrt{3}}$
R.	The distance between the foci of H is	3.	$\frac{2}{\sqrt{3}}$
S.	The length of the latus rectum of H is	4.	4

The correct option is:

- (a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
 (b) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 2$
 (c) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 2$
 (d) $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 1$

18. Let $f_1: R \rightarrow R, f_2: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R,$

$f_3: \left(-1, e^2 - 2\right) \rightarrow R$, and $f_4: R \rightarrow R$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right).$

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse

trigonometric function $\tan^{-1}x$ assumes values in

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right),$

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where, for $t \in R, [t]$ denotes the greatest integer less than or equal to t ,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$

List-I		List-II	
P.	The function f_1 is	1.	NOT continuous at $x = 0$
Q.	The function f_2 is	2.	continuous at $x = 0$ and NOT differentiable at $x = 0$
R.	The function f_3 is	3.	differentiable at $x = 0$ and its derivative is NOT continuous at $x = 0$
S.	The function f_4 is	4.	differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is:

- (a) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$
 (b) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3$
 (c) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
 (d) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3$

SOLUTIONS

PAPER-1

1. (a, b, d) : $\arg(-1-i) = -3\pi/4$

$$\text{We have, } f(t) = \arg|-1+it| = \begin{cases} \pi - \tan^{-1} t, & t \geq 0 \\ -\pi + \tan^{-1} t, & t < 0 \end{cases}$$

$$\lim_{t \rightarrow 0^+} f(t) = \pi, \quad \lim_{t \rightarrow 0^-} f(t) = -\pi$$

Hence f is not continuous at 0.

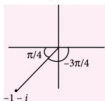
$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$= (\arg z_1 - \arg z_2) + 2n\pi - \arg z_1 + \arg z_2 = 2n\pi, n \in \mathbb{Z}$$

$$\text{Now, } \arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$$

$$\therefore \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \text{ is real.}$$

$$\Rightarrow z_1, z_2, z_3 \text{ are concyclic.}$$



[Rating : Doable]

2. (b, c, d) : Using cosine rule,

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{(10\sqrt{3})^2 + 10^2 - PR^2}{2 \cdot 10 \cdot 10\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cdot 2 \times 10\sqrt{3} \times 10 = 400 - PR^2$$

$$\Rightarrow 300 = 400 - PR^2 \Rightarrow PR^2 = 100 \quad \therefore PR = 10$$

$$\text{So, } QR = PR \quad \therefore \angle QPR = 30^\circ$$

$$\text{ar}(\Delta PQR) = \frac{1}{2} \cdot 10\sqrt{3} \cdot 10 \sin 30^\circ = \frac{1}{2} \cdot 10 \cdot 10\sqrt{3} \cdot \frac{1}{2} = 25\sqrt{3}$$

$$\text{Also, } \angle QRP = 180^\circ - 2 \times 30^\circ = 120^\circ$$

$$\text{Now, } r = \frac{\Delta}{s} = \frac{25\sqrt{3}}{\frac{10+10+10\sqrt{3}}{2}} = \frac{25\sqrt{3}}{10+5\sqrt{3}} = \frac{25\sqrt{3}}{5(2+\sqrt{3})}$$

$$= 5\sqrt{3}(2-\sqrt{3}) = 10\sqrt{3} - 15$$

$$\text{Also, } R = \frac{10}{2 \sin 30^\circ} = 10$$

$$\text{Area} = \pi R^2 = 100\pi$$

[Rating : Easy]

3. (c, d) : Let DR's of line of intersection of P_1 and P_2 be (a, b, c) .

$$\text{Then, } 2a + b - c = 0, \quad a + 2b + c = 0$$

$$\Rightarrow \frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1} \Rightarrow \frac{a}{3} = \frac{b}{-3} = \frac{c}{3}$$

$$\text{So, the DR's are } (1, -1, 1).$$

The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ can be rewritten as

$$\frac{x-\frac{4}{3}}{3} = \frac{y-\frac{1}{3}}{-3} = \frac{z}{3}$$

So, the line is parallel to the line of intersection of plane P_1 and P_2 .

The angle between P_1 and P_2

$$= \cos^{-1} \left(\frac{2 \cdot 1 + 1 \cdot 2 - 1 \cdot 1}{\sqrt{6}\sqrt{6}} \right) = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$$

The plane P_3 is given by

$$1(x-4) - 1(y-2) + 1(z+2) = 0$$

$$\text{i.e., } x - y + z = 0$$

$$\text{Distance of } (2, 1, 1) \text{ from the plane } P_3 = \frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

[Rating : Easy]

4. (a, b, d) : As f is continuous and it is not constant, we must have an interval (r, s) on which f is one-one. Applying L.M.V.T. on $(-4, 0)$, we have

$$\frac{f(0) - f(-4)}{4} = f'(c), \quad \text{where } c \in (-4, 0)$$

$$\Rightarrow |f'(c)| \leq \left| \frac{2+2}{4} \right| = 1$$

$\lim_{x \rightarrow \infty} f(x) = 1$ is incorrect as a counter example.

$f(x) = \sin(\sqrt{85}x)$ satisfies the given conditions

but $\lim_{x \rightarrow \infty} \sin(\sqrt{85}x)$ doesn't exist.

Consider $A(x) = f^2(x) + f'^2(x)$

$$A(0) = 85$$

$$\Rightarrow c \in (-4, 0) \text{ such that } |f'(c)|^2 \leq 1$$

$$A(c) = f^2(c) + (f'(c))^2$$

$$\leq 4 + 1 = 5 \text{ then } c \in (-4, 0)$$

Similarly, let d be such that $A(d) \leq 5$ then $d \in (0, 4)$

$$A(0) = 85$$

$\therefore A(x)$ must have maxima in (c, d) , say at β

$$\Rightarrow A'(\beta) = 0 \text{ and } A(\beta) \geq 85$$

$$\text{Now, } 2f'(\beta) [f(\beta) + f''(\beta)] = 0$$

$$f'(\beta) = 0 \text{ is not possible, hence } f(\beta) + f''(\beta) = 0.$$

[Rating : Challenging]

5. (b, c) : We have $f'(x) = e^{(f(x)-g(x))} g'(x)$

$$\Rightarrow e^{-f(x)} f'(x) = e^{-g(x)} g'(x)$$

Integrating we get, $-e^{-f(x)} = -e^{-g(x)} + \lambda$

$$\text{i.e., } e^{-g(x)} - e^{-f(x)} = \lambda$$

Put $x = 1$ and $x = 2$ respectively, we get

$$e^{-g(1)} - e^{-f(1)} = \lambda \text{ and } e^{-g(2)} - e^{-f(2)} = \lambda$$

We have $e^{-g(1)} - e^{-f(1)} = e^{-g(2)} - e^{-f(2)}$

$$\Rightarrow e^{-g(1)} - e^{-1} = e^{-1} - e^{-f(2)} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2/e$$

$$\Rightarrow e^{-f(2)} < \frac{2}{e} \Rightarrow -f(2) < \ln 2 - 1$$

$$\therefore f(2) > 1 - \ln 2$$

Similar things holds for $g(1)$.

[Rating : Doable]

$$6. \text{ (b, c) : } f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt \quad \dots(i)$$

$$\Rightarrow e^{-x} f(x) = e^{-x} (1 - 2x) + \int_0^x e^{-t} f(t) dt$$

Differentiating w.r.t. x , we get

$$e^{-x} f'(x) - e^{-x} f(x) = e^{-x} (-2) - e^{-x} (1 - 2x) + e^{-x} f(x)$$

This simplifies to

$$f'(x) - 2f(x) = 2x - 3 \text{ (Linear D.E.)}$$

$$\therefore \text{I.F.} = e^{-2x}$$

$$\therefore \text{The required solution is } f(x)e^{-2x} = \int (2x-3)e^{-2x} dx$$

$$= (2x-3) \cdot \frac{e^{-2x}}{-2} + 2 \int \frac{e^{-2x}}{2} dx + \lambda$$

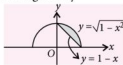
$$= \frac{-(2x-3)}{2} e^{-2x} - \frac{1}{2} e^{-2x} + \lambda$$

$$\therefore y = f(x) = (1-x) + \lambda e^{2x}$$

$$\text{Put } x = 0 \text{ in (i)} \Rightarrow y = 1 \Rightarrow 1 = 1 + \lambda \therefore \lambda = 0$$

$$\therefore y = 1 - x$$

The required area is given by



$$\text{Required area} = \frac{\pi}{2} \cdot \frac{1}{2} - 1 + \frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}$$

[Rating : Easy]

$$7. \text{ (8) : } ((\log_2 9)^2)^{\frac{1}{\log_2 (\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$$

$$= (\log_2 9)^{\frac{2}{\log_2 (\log_2 9)}} \cdot (\sqrt{7})^{\log_4 7}$$

$$= (\log_2 9)^{2 \log_2 \log_2 9 \cdot (2)} \cdot 7^{\frac{1}{2} \log_7 4} = 4 \cdot 2 = 8$$

[Rating : Doable]

8. (625) : The last two digits can be 12, 24, 32, 44, 52 for the number to be divisible by 4

The number of ways = $5 \cdot 5 \cdot 5 \cdot 5 = 625$

[Rating : Easy]

9. (3748) : Here $X = \{1, 6, 11, \dots, 10086\}$

and $Y = \{9, 16, 23, \dots, 14128\}$

The intersection of X and Y is an A.P. with 16 as first term and 35 as common difference.

The series becomes 16, 51, 86, ...

Now, k^{th} term = $16 + (k-1) \cdot 35 \leq 10086$

$$\text{i.e. } k \leq \frac{10105}{35} \therefore k \leq 288 \text{ (As } k \text{ is to be an integer)}$$

$$\text{Hence, } n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$= 2018 + 2018 - 288 = 3748$$

[Rating : Easy]

$$10. (2) : \text{Let } f(x) = \sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i$$

$$= (x^2 + x^3 + \dots \text{to } \infty) - x \left(\frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots \text{to } \infty \right)$$

$$= \frac{x^2}{1-x} - \frac{x \cdot \frac{x}{2}}{1 - \frac{x}{2}} = \frac{x^2}{1-x} - \frac{x^2}{2-x}$$

$$\text{Again, let } g(x) = \sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i$$

$$= \left[\left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 + \dots \text{to } \infty \right] - [(-x) + (-x)^2 + \dots \text{to } \infty]$$

$$= \left[-\frac{x}{2} - \frac{-x}{1+x} \right] - \left[\frac{x}{1+x} - \frac{x}{2+x} \right]$$

The given equation becomes

$$\sin^{-1} f(x) + \cos^{-1} g(x) = \pi/2$$

So we must have $f(x) = g(x)$

$$\Rightarrow \frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{x}{1+x} - \frac{x}{2+x}$$

$$\Rightarrow \frac{x^2}{(1-x)(2-x)} = \frac{x}{(2+x)(1+x)}$$

$$\Rightarrow x = 0 \text{ and } x(2+x)(1+x) = (1-x)(2-x)$$

$$\text{Let } h(x) = x(2+x)(1+x) - (1-x)(2-x)$$

$$= x^3 + 2x^2 + 5x - 2$$

$$\begin{aligned}\text{Now, } h\left(-\frac{1}{2}\right) &= -\frac{1}{2}\left(2 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right) - \left(1 + \frac{1}{2}\right)\left(2 + \frac{1}{2}\right) \\ &= -\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} - \frac{3}{2} \cdot \frac{5}{2} = \frac{3}{2}\left(-\frac{1}{4} - \frac{5}{2}\right) < 0 \\ h\left(\frac{1}{2}\right) &= \frac{1}{2}\left(2 + \frac{1}{2}\right)\left(1 + \frac{1}{2}\right) - \left(1 - \frac{1}{2}\right)\left(2 - \frac{1}{2}\right) \\ &= \frac{1}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} - \frac{1}{2} \cdot \frac{3}{2} > 0\end{aligned}$$

$\therefore \exists$ a root between $-\frac{1}{2}$ and $\frac{1}{2}$.

Also, $h'(x) = 3x^2 + 4x + 5 > 0$

$\Rightarrow h(x)$ has exactly one real root in $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

[Rating : Doable]

11. (1) : We have

$$\log y_n = \frac{1}{n} \left\{ \log \left(1 + \frac{1}{n}\right) + \log \left(1 + \frac{2}{n}\right) + \dots + \log \left(1 + \frac{x}{n}\right) \right\}$$

$$\begin{aligned}\therefore \lim_{n \rightarrow \infty} \log y_n &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \left(1 + \frac{r}{n}\right) \\ &= \int_0^1 \log(1+x) dx = \int_1^2 \log x dx \\ &= [x \log x - x]_1^2 = 2 \log 2 - 1 = \log \frac{4}{e}\end{aligned}$$

$$\therefore L = 4/e \Rightarrow [L] = 1.$$

[Rating : Easy]

12. (3) : Here $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$

Also, $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = 2 \cos \alpha$

We have, $\vec{c} \cdot \vec{a} = x\vec{a} \cdot \vec{a} + 0 + 0$

$\therefore x = \vec{c} \cdot \vec{a} = 2 \cos \alpha$ Similarly, $y = 2 \cos \alpha$

Now, $c^2 = x^2 + y^2 + 1$

We have $4 = 2(4 \cos^2 \alpha) + 1 \Rightarrow 3 = 8 \cos^2 \alpha$

[Rating : Doable]

13. (0.5) : As α and β are the roots of the given equation, we have

$$\sqrt{3} a \cos \alpha + 2b \sin \alpha = c \quad \dots(i)$$

$$\sqrt{3} a \cos \beta + 2b \sin \beta = c \quad \dots(ii)$$

On subtracting (ii) from (i), we get,

$$\sqrt{3}(\cos \alpha - \cos \beta) + \frac{2b}{a}(\sin \alpha - \sin \beta) = 0$$

$$\Rightarrow \sqrt{3} \left(-2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} \right) + \frac{2b}{a} 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} = 0$$

$$\Rightarrow -\sqrt{3} \sin \frac{\pi}{6} + \frac{2b}{a} \cos \frac{\pi}{6} = 0$$

$$\Rightarrow -\sqrt{3} \cdot \frac{1}{2} + \frac{2b}{a} \cdot \frac{\sqrt{3}}{2} = 0 \Rightarrow -1 + \frac{2b}{a} = 0 \therefore \frac{b}{a} = \frac{1}{2} = 0.5$$

[Rating : Doable]

14. (4)

15. (a)

16. (d)

17. (a)

18. (c)

PAPER-2

1. (d) 2. (b, d) 3. (a, d) 4. (a, c)

5. (a, c, d) 6. (b, c, d) 7. (2) 8. (4)

9. (119) 10. (0.4) 11. (2) 12. (8)

13. (0.5) 14. (646) 15. (a) 16. (c)

17. (b) 18. (d)

For complete solutions refer to
MTG JEE Advanced Explorer

SAMURAI SUDOKU

SOLUTION · JUNE 2018



1	3	5	9	4	2	8	6	7		2	5	9	3	7	8	1	6	4
9	8	2	6	7	5	3	1	4		4	7	3	6	1	2	5	8	9
6	7	4	8	3	1	2	9	5		8	6	1	9	5	4	7	2	3
5	4	6	2	8	9	7	3	1		9	8	2	1	6	3	4	7	5
3	2	8	7	1	6	5	4	9		5	4	6	7	8	9	2	3	1
7	1	9	4	5	3	6	2	8		3	1	7	4	2	5	6	9	8
8	6	3	1	9	7	4	5	2	1	7	9	6	3	8	5	4	7	9
2	9	7	5	6	4	1	8	3	2	6	5	7	9	4	2	3	1	8
4	5	1	3	2	8	9	7	6	8	4	3	1	2	5	8	9	6	3
						8	6	5	3	9	7	4	1	2				
						3	4	9	5	2	1	8	6	7				
						2	1	7	6	8	4	9	5	3				
1	8	9	7	2	6	5	3	4	7	1	6	2	8	9	5	1	3	4
2	7	4	3	1	5	6	9	8	4	3	2	5	7	1	4	2	6	3
6	3	5	8	4	9	7	2	1	9	5	8	3	4	6	7	9	8	1
4	5	2	1	3	8	9	7	6						9	2	8	1	5
8	9	3	6	7	4	2	1	5						4	1	3	6	7
7	1	6	9	5	2	4	8	3						6	5	7	8	3
9	2	8	4	6	1	3	5	7						8	3	4	2	6
3	8	1	5	9	7	8	4	2						7	9	5	3	8
5	4	7	2	8	3	1	6	9						1	6	2	9	4

MHT CET

1. If $\int_0^K \frac{dx}{2+18x^2} = \frac{\pi}{24}$, then the value of K is

- (a) 3 (b) 4
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. The cartesian co-ordinates of the point on the parabola $y^2 = -16x$, whose parameter is $\frac{1}{2}$, are

- (a) $(-2, 4)$ (b) $(4, -1)$
(c) $(-1, -4)$ (d) $(-1, 4)$

3. $\int \frac{1}{\sin x \cdot \cos^3 x} dx =$

- (a) $\sec x + \log |\sec x + \tan x| + c$
(b) $\sec x - \tan x + c$
(c) $\sec x + \log |\sec x - \tan x| + c$
(d) $\sec x + \log |\operatorname{cosec} x - \cot x| + c$

4. If $\log_{10} \left(\frac{x^3 - y^3}{x^3 + y^3} \right) = 2$, then $\frac{dy}{dx} =$

- (a) $\frac{x}{y}$ (b) $-\frac{y}{x}$ (c) $-\frac{x}{y}$ (d) $\frac{y}{x}$

5. If $f: R - \{2\} \rightarrow R$ is a function defined by

$$f(x) = \frac{x^2 - 4}{x - 2}, \text{ then its range is}$$

- (a) R (b) $R - \{2\}$
(c) $R - \{4\}$ (d) $R - \{-2, 2\}$

6. If $f(x) = x^2 + \alpha$ for $x \geq 0$

$$= 2\sqrt{x^2 + 1} + \beta \text{ for } x < 0$$

is continuous at $x = 0$ and $f\left(\frac{1}{2}\right) = 2$, then $\alpha^2 + \beta^2$ is

- (a) 3 (b) $\frac{8}{25}$ (c) $\frac{25}{8}$ (d) $\frac{1}{3}$

7. If $y = (\tan^{-1} x)^2$, then $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} =$

- (a) 4 (b) 2
(c) 1 (d) 0

8. The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - kx - 2y + 2 = 0$, then the value of k is

- (a) -11 (b) 31 (c) 11 (d) -31

9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then $(A^2 - 5A)A^{-1} =$

(a) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -1 & 1 \\ 2 & -4 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -2 & 1 \\ 4 & -2 & -3 \\ 1 & 4 & -2 \end{bmatrix}$

10. The equation of line passing through $(3, -1, 2)$ and perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ is

(a) $\frac{x+3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(b) $\frac{x-3}{3} = \frac{y+1}{2} = \frac{z-2}{2}$

(c) $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(d) $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-2}{3}$

11. Letters in the word HULULULU are rearranged. The probability of all three L being together is
 (a) $\frac{3}{20}$ (b) $\frac{2}{5}$ (c) $\frac{3}{28}$ (d) $\frac{5}{23}$
12. The sum of the first 10 terms of the series $9 + 99 + 999 + \dots$, is
 (a) $\frac{9}{8}(9^{10} - 1)$ (b) $\frac{100}{9}(10^9 - 1)$
 (c) $10^9 - 1$ (d) $\frac{100}{9}(10^{10} - 1)$
13. If A, B, C are the angles of $\triangle ABC$, then $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A =$
 (a) 0 (b) 1
 (c) 2 (d) -1
14. If $\int \frac{dx}{\sqrt{16-9x^2}} = A \sin^{-1}(Bx) + C$, then $A + B =$
 (a) $\frac{9}{4}$ (b) $\frac{19}{4}$ (c) $\frac{3}{4}$ (d) $\frac{13}{12}$
15. $\int e^x \left[\frac{2 + \sin 2x}{1 + \cos 2x} \right] dx =$
 (a) $e^x \tan x + c$ (b) $e^x + \tan x + c$
 (c) $2e^x \tan x + c$ (d) $e^x \tan 2x + c$
16. A coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, then $P(X = 1) =$
 (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{3}{4}$
17. If $2 \sin \left(\theta + \frac{\pi}{3} \right) = \cos \left(\theta - \frac{\pi}{6} \right)$, then $\tan \theta =$
 (a) $\sqrt{3}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$
18. The area of the region bounded by $x^2 = 4y$, $y = 1$, $y = 4$ and the y-axis lying in the first quadrant is _____ square units.
 (a) $\frac{22}{3}$ (b) $\frac{28}{3}$ (c) 30 (d) $\frac{21}{4}$
19. If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, for $x \neq 0$ is continuous at $x = 0$, then value of $f(0)$ is
 (a) $\frac{2}{3}$ (b) $\frac{5}{2}$ (c) 1 (d) $\frac{3}{2}$
20. The maximum value of $2x + y$ subject to $3x + 5y \leq 26$ and $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$ is
 (a) 12 (b) 11.5 (c) 10 (d) 17.33
21. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors having magnitudes 1, 2, 3 respectively, then $[\vec{a} + \vec{b} + \vec{c}, \vec{b} - \vec{a}, \vec{c}] =$
 (a) 0 (b) 6 (c) 12 (d) 18
22. If points $P(4, 5, x)$, $Q(3, y, 4)$ and $R(5, 8, 0)$ are collinear, then the value of $x + y$ is
 (a) -4 (b) 3 (c) 5 (d) 4
23. If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is two times the other, then
 (a) $8h^2 = 9ab$ (b) $8h^2 = 9ab^2$
 (c) $8h = 9ab$ (d) $8h = 9ab^2$
24. The equation of the line passing through the point $(-3, 1)$ and bisecting the angle between co-ordinate axes is
 (a) $x + y + 2 = 0$ (b) $-x + y + 2 = 0$
 (c) $x - y + 4 = 0$ (d) $2x + y + 5 = 0$
25. The negation of the statement: "Getting above 95% marks is necessary condition for Hema to get the admission in good college".
 (a) Hema gets above 95% marks but she does not get the admission in good college
 (b) Hema does not get above 95% marks and she gets admission in good college
 (c) If Hema does not get above 95% marks then she will not get the admission in good college
 (d) Hema does not get above 95% marks or she gets the admission in good college
26. $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 179^\circ =$
 (a) 0 (b) 1 (c) $-\frac{1}{2}$ (d) -1
27. If planes $cx - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through straight line, then $a^2 + b^2 + c^2 =$
 (a) $1 - abc$ (b) $abc - 1$
 (c) $1 - 2abc$ (d) $2abc - 1$
28. The point of intersection of lines represented by $x^2 - y^2 + x + 3y - 2 = 0$ is
 (a) (1, 0) (b) (0, 2)
 (c) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$
29. A die is rolled. If X denotes the number of positive divisors of the outcome then the range of the random variable X is
 (a) {1, 2, 3} (b) {1, 2, 3, 4}
 (c) {1, 2, 3, 4, 5, 6} (d) {1, 3, 5}

30. A die is thrown four times. The probability of getting perfect square in at least one throw is

(a) $\frac{16}{81}$ (b) $\frac{65}{81}$ (c) $\frac{23}{81}$ (d) $\frac{58}{81}$

31. $\int_0^{\pi/4} x \cdot \sec^2 x \, dx =$

(a) $\frac{\pi}{4} + \log \sqrt{2}$ (b) $\frac{\pi}{4} - \log \sqrt{2}$

(c) $1 + \log \sqrt{2}$ (d) $1 - \frac{1}{2} \log 2$

32. In ΔABC , with usual notations, if a, b, c are in A.P.,

then $a \cos^2 \left(\frac{C}{2} \right) + c \cos^2 \left(\frac{A}{2} \right) =$

(a) $\frac{3a}{2}$ (b) $\frac{3c}{2}$ (c) $\frac{3b}{2}$ (d) $\frac{3abc}{2}$

33. If $x = e^{\theta}(\sin \theta - \cos \theta)$, $y = e^{\theta}(\sin \theta + \cos \theta)$, then $\frac{dy}{dx}$

at $\theta = \frac{\pi}{4}$ is

(a) 1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

34. The number of solutions of $\sin x + \sin 3x + \sin 5x = 0$

in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$ is

(a) 2 (b) 3 (c) 4 (d) 5

35. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, then $x =$

(a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

36. Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then the value of $a_{31} A_{31} +$

$a_{32} A_{32} + a_{33} A_{33}$ is

(a) 1 (b) 13 (c) -1 (d) -13

37. The contrapositive of the statement: "If the weather is fine then my friends will come and we go for a picnic."

(a) The weather is fine but my friends will not come or we do not go for a picnic

(b) If my friends do not come or we do not go for picnic then weather will not be fine

(c) If the weather is not fine then my friends will not come or we do not go for a picnic

- (d) The weather is not fine but my friends will come and we go for a picnic

38. If $f(x) = \frac{x}{x^2 + 1}$ is increasing function then the value of x lies in

(a) R (b) $(-\infty, -1)$

(c) $(1, \infty)$ (d) $(-1, 1)$

39. If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n-1) : n \in \mathbb{N}\}$, then $X \cap Y =$

(a) X (b) Y (c) ϕ (d) $\{0\}$

40. The statement pattern $p \wedge (\sim p \wedge q)$ is

(a) a tautology

(b) a contradiction

(c) equivalent to $p \wedge q$

(d) equivalent to $p \vee q$

41. If the line $y = 4x - 5$ touches to the curve $y^2 = ax^3 + b$ at the point $(2, 3)$, then $7a + 2b =$

(a) 0 (b) 1 (c) -1 (d) 2

42. The sides of a rectangle are given by $x = \pm a$ and $y = \pm b$. The equation of the circle passing through the vertices of the rectangle is

(a) $x^2 + y^2 = a^2$

(b) $x^2 + y^2 = a^2 + b^2$

(c) $x^2 + y^2 = a^2 - b^2$

(d) $(x-a)^2 + (y-b)^2 = a^2 + b^2$

43. The minimum value of the function $f(x) = x \log x$ is

(a) $-\frac{1}{e}$ (b) $-e$ (c) $\frac{1}{e}$ (d) e

44. If $X \sim B(n, p)$ with $n = 10, p = 0.4$ then $E(X^2) =$

(a) 4 (b) 2.4 (c) 3.6 (d) 18.4

45. The general solution of differential equation

$\frac{dx}{dy} = \cos(x+y)$ is

(a) $\tan\left(\frac{x+y}{2}\right) = y+c$ (b) $\tan\left(\frac{x+y}{2}\right) = x+c$

(c) $\cot\left(\frac{x+y}{2}\right) = y+c$ (d) $\cot\left(\frac{x+y}{2}\right) = x+c$

46. If planes $\vec{r} \cdot (p\hat{i} - \hat{j} + 2\hat{k}) + 3 = 0$ and

$\vec{r} \cdot (2\hat{i} - p\hat{j} - \hat{k}) - 5 = 0$ include angle $\frac{\pi}{3}$, then the value of p is

(a) 1, -3 (b) -1, 3

(c) -3 (d) 3

47. The order of the differential equation of all parabolas, whose latus rectum is $4a$ and axis parallel to the x -axis, is
 (a) one (b) four
 (c) three (d) two
48. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $x-3 = \frac{y-k}{2} = z$ intersect then the value of k is
 (a) $\frac{9}{2}$ (b) $\frac{1}{2}$
 (c) $\frac{5}{2}$ (d) $\frac{7}{2}$
49. If a line makes angles 120° and 60° with the positive directions of X and Z axes respectively then the angle made by the line with positive Y -axis is
 (a) 150° (b) 60°
 (c) 135° (d) 120°
50. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$ respectively. The position vector of the point N which divides the line segment LM in the ratio $2 : 1$ externally is
 (a) $3\vec{b}$ (b) $4\vec{b}$
 (c) $5\vec{b}$ (d) $3\vec{a} + 4\vec{b}$

SOLUTIONS

1. (c): Let $I = \int_0^K \frac{dx}{2+18x^2} = \frac{1}{18} \int_0^K \frac{dx}{\frac{2}{9} + x^2}$

$$\Rightarrow I = \frac{1}{18} \int_0^K \frac{dx}{\frac{1}{9} + x^2} = \frac{1}{18} \int_0^K \frac{dx}{\left(\frac{1}{3}\right)^2 + x^2}$$

$$\Rightarrow I = \frac{1}{18} \left[\frac{1}{\left(\frac{1}{3}\right)} \tan^{-1} \left(\frac{x}{\left(\frac{1}{3}\right)} \right) \right]_0^K = \frac{\pi}{24}$$

$$\Rightarrow \tan^{-1} 3K = \frac{\pi}{4} = \tan^{-1} 1 \Rightarrow 3K = 1 \Rightarrow K = \frac{1}{3}.$$

2. (d): We have, $y^2 = -16x$

Compare it with $y^2 = -4ax$, we get $a = 4$

$\therefore P(t) = (-at^2, 2at)$, where $t = \frac{1}{2}$

$\therefore P = \left(-4 \times \frac{1}{4}, 2 \times \frac{1}{2} \times 4 \right)$ i.e., $(-1, 4)$

3. (d): Let $I = \int \frac{1}{\sin x \cos^2 x} dx$

$$= \int \frac{\cos^2 x + \sin^2 x}{\sin x \cos^2 x} dx = \int \frac{1}{\sin x} dx + \int \frac{\sin x}{\cos^2 x} dx$$

$$= \int \operatorname{cosec} x dx + \int \tan x \sec x dx$$

$$= \log |\operatorname{cosec} x - \cot x| + \sec x + c$$

4. (d): We have, $\log_{10} \left(\frac{x^3 - y^3}{x^3 + y^3} \right) = 2$

$$\Rightarrow \log_{10} (x^3 - y^3) - \log_{10} (x^3 + y^3) = 2$$

$$\Rightarrow \log (x^3 - y^3) - \log (x^3 + y^3) = 2 \log 10$$

On differentiating w.r.t. x , we get

$$\frac{1}{x^3 - y^3} \left(3x^2 - 3y^2 \frac{dy}{dx} \right) - \frac{1}{x^3 + y^3} \left(3x^2 + 3y^2 \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \frac{3x^2}{x^3 - y^3} - \frac{3x^2}{x^3 + y^3} - \left(\frac{3y^2}{x^3 - y^3} + \frac{3y^2}{x^3 + y^3} \right) \frac{dy}{dx} = 0$$

$$\Rightarrow 3y^2 \left[\frac{x^3 + y^3 + x^3 - y^3}{(x^3 - y^3)(x^3 + y^3)} \right] \frac{dy}{dx}$$

$$= 3x^2 \left[\frac{x^3 + y^3 - x^3 + y^3}{(x^3 - y^3)(x^3 + y^3)} \right]$$

$$\Rightarrow (3y^2 \times 2x^3) \frac{dy}{dx} = 3x^2 \times 2y^3 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

5. (c): We have, $y = f(x) = \frac{x^2 - 4}{x - 2}, x \neq 2$

$$= \frac{(x-2)(x+2)}{x-2}, x \neq 2 = x+2, x \neq 2.$$

It follows from the above relation that y take all real values except when x takes values in the set $R - \{2\}$

$\therefore \text{Range}(f) = R - \{4\}$

6. (c): We have $f(x) = \begin{cases} x^2 + \alpha & \text{for } x \geq 0 \\ 2\sqrt{x^2 + 1} + \beta & \text{for } x < 0 \end{cases}$

$\therefore \text{L.H.L. (at } x=0) = \lim_{h \rightarrow 0} 2\sqrt{(0-h)^2 + 1} + \beta$

$$= \lim_{h \rightarrow 0} 2\sqrt{h^2 + 1} + \beta = 2 + \beta$$

and R.H.L. (at $x=0$) = $\lim_{h \rightarrow 0} (0+h)^2 + \alpha = \lim_{h \rightarrow 0} h^2 + \alpha = \alpha$

Now, $f(0) = (0)^2 + \alpha = \alpha$

Since $f(x)$ is continuous at $x = 0$.

$\Rightarrow$ L.H.L. at $x = 0 =$ R.H.L. at $x = 0 = f(0)$

$\Rightarrow 2 + \beta = \alpha$... (i)

$$\text{Also } f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \alpha \Rightarrow \frac{1}{4} + \alpha = 2 \Rightarrow \alpha = \frac{7}{4}.$$

$$\text{Put } \alpha = \frac{7}{4} \text{ in (i), we get } 2 + \beta = \frac{7}{4} \Rightarrow \beta = \frac{-1}{4}$$

$$\text{Now, } \alpha^2 + \beta^2 = \left(\frac{7}{4}\right)^2 + \left(\frac{-1}{4}\right)^2 = \frac{49}{16} + \frac{1}{16} = \frac{25}{8}$$

7. (b) : We have, $y = (\tan^{-1} x)^2$

$$\Rightarrow \frac{dy}{dx} = 2 \tan^{-1} x \times \frac{1}{1+x^2} \quad \dots (i)$$

$$\begin{aligned} \text{Now, } \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left(\frac{2 \tan^{-1} x}{1+x^2} \right) \\ &= \frac{(1+x^2) \left(\frac{2}{1+x^2} \right) - 2 \tan^{-1} x \times 2x}{(1+x^2)^2} \end{aligned}$$

$$\Rightarrow \frac{d^2 y}{dx^2} (1+x^2)^2 = 2 - 4x \tan^{-1} x = 2 - 4x \left[\frac{dy}{dx} \frac{(1+x^2)}{2} \right] \quad [\text{using (i)}]$$

$$= 2 - 2x(1+x^2) \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2 y}{dx^2} (1+x^2)^2 + 2x(x^2+1) \frac{dy}{dx} = 2$$

8. (c) : We have, $5x^2 + xy - kx - 2y + 2 = 0$

As y^2 is absent in given equation, let the equation of second line be $ax + c = 0$.

$$\Rightarrow (5x + y - 1)(ax + c) = 0$$

$$\Rightarrow 5ax^2 + 5cx + axy + cy - ax - c = 0$$

Given equation is $5x^2 + xy - kx - 2y + 2 = 0$

$$\therefore a = 1, c = -2 \text{ and } -k = 5c - a \Rightarrow -k = 5(-2) - 1 \Rightarrow k = 11$$

$$9. (b) : \text{ We have, } A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$|A| = 1(4-4) - 2(-4-2) + 3(-2-1) = 12 - 9 = 3 \neq 0$$

$\Rightarrow A^{-1}$ exists.

$$\text{Now, } (A^2 - 5A) A^{-1} = A^2 A^{-1} - 5A A^{-1} = A - 5I$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

10. (c) : The required line is perpendicular to the lines which are parallel to the vectors $\vec{b}_1 = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b}_2 = \hat{i} - 2\hat{j} + 2\hat{k}$. So, it is parallel to the vector $\vec{b} = \vec{b}_1 \times \vec{b}_2$

$$\text{Now, } \vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= (-4+2)\hat{i} - (4-1)\hat{j} + (-4+2)\hat{k} = -2\hat{i} - 3\hat{j} - 2\hat{k}$$

Thus, the required line passes through the point $(3, -1, 2)$

and is parallel to the vector $\vec{b} = -2\hat{i} - 3\hat{j} - 2\hat{k}$

$$\therefore \text{ Equation of line is } \frac{x-3}{-2} = \frac{y+1}{-3} = \frac{z-2}{-2}$$

11. (c) : In the word 'HULULULU', there are 8 letters in which there are 4U's, 3L's and 1H.

$$\therefore \text{ Total number of arrangements} = \frac{8!}{3!4!} = 280$$

We take all 3 L's together and treating them as one letter.

Now, the number of words in which 3L's come together

$$= \frac{6!}{4!} = 30$$

$$\therefore \text{ Required probability} = \frac{30}{280} = \frac{3}{28}$$

12. (b) : Let $S = 9 + 99 + 999 + \dots$ up to 10 terms.

$$= (10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ (up to 10 terms)}$$

$$= (10 + 100 + 1000 + \dots \text{ (up to 10 terms)}) - (1 + 1 + \dots \text{ 10 times})$$

$$= (10^1 + 10^2 + 10^3 + \dots + 10^{10}) - (1 + 1 + \dots \text{ 10 times})$$

$$= \frac{10(10^{10} - 1)}{10 - 1} - 10 = \frac{10}{9}(10^{10} - 1) - 10$$

$$= \frac{10 \times 10^{10} - 10 - 10 \times 9}{9} = \frac{10 \times 10^{10} - 100}{9} = \frac{100}{9} [10^9 - 1]$$

13. (b) : $\therefore A + B + C = \pi \Rightarrow A + B = \pi - C$

Now, $\cot(A + B) = \cot(\pi - C)$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$$

$$\Rightarrow \cot A \cot B - 1 = -\cot A \cot C - \cot B \cot C$$

$$\Rightarrow \cot A \cot B + \cot B \cot C + \cot A \cot C = 1.$$

$$14. (d) : \text{ Let } I = \int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{(4/3)^2 - x^2}}$$

$$= \frac{1}{3} \sin^{-1} \frac{x}{(4/3)} + C = \frac{1}{3} \sin^{-1} \left(\frac{3x}{4} \right) + C$$

On comparing above equation with $A \sin^{-1}(Bx) + C$, we get

$$A = \frac{1}{3}, B = \frac{3}{4} \Rightarrow A + B = \frac{1}{3} + \frac{3}{4} = \frac{13}{12}$$

$$\begin{aligned}
 15. \quad (a): \text{ Let } I &= \int e^x \left[\frac{2 + \sin 2x}{1 + \cos 2x} \right] dx \\
 &= \int e^x \left[\frac{2 + 2 \sin x \cos x}{2 \cos^2 x} \right] dx = \int e^x \left[\frac{1}{\cos^2 x} + \frac{\sin x}{\cos x} \right] dx \\
 &= \int e^x \left[\tan x + \sec^2 x \right] dx \\
 &= e^x \tan x + c \quad \left[\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c \right]
 \end{aligned}$$

16. (d) : Total number of outcomes = 8 (HHH, HTH, THH, HHT, HTT, THT, TTH, TTT)
Favorable outcomes = {HTH, THH, HHT, HTT, THT, TTH} which are 6 in number.

$$\therefore \text{Required probability} = \frac{6}{8} = \frac{3}{4}.$$

$$\begin{aligned}
 17. \quad (d) : \text{ We have, } 2 \sin \left(\theta + \frac{\pi}{3} \right) &= \cos \left(\theta - \frac{\pi}{6} \right) \\
 \Rightarrow 2 \left(\sin \theta \cos \frac{\pi}{3} + \cos \theta \sin \frac{\pi}{3} \right) &= \cos \theta \cos \frac{\pi}{6} + \sin \theta \sin \frac{\pi}{6} \\
 \Rightarrow 2 \left(\sin \theta \times \frac{1}{2} + \cos \theta \times \frac{\sqrt{3}}{2} \right) &= \cos \theta \times \frac{\sqrt{3}}{2} + \sin \theta \times \frac{1}{2} \\
 \Rightarrow \frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta &= 0 \Rightarrow \frac{1}{2} \sin \theta = -\frac{\sqrt{3}}{2} \cos \theta \\
 \Rightarrow \tan \theta &= -\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad (b) : \text{ Area bounded by curve } x^2 = 4y \text{ or } x = 2\sqrt{y} \\
 \text{between } y = 1 \text{ and } y = 4 \text{ is } \int_1^4 2\sqrt{y} \, dy \\
 = 2 \left[\frac{y^{3/2}}{3/2} \right]_1^4 = \frac{28}{3} \text{ sq. units}
 \end{aligned}$$

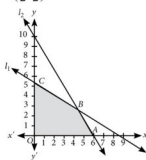

$$\begin{aligned}
 19. \quad (d) : \text{ Given, } f(x) &= \frac{e^{x^2} - \cos x}{x^2} \text{ is continuous at } x = 0 \\
 \Rightarrow \text{L.H.L.} &= \text{R.H.L.} = f(0) \text{ at } x = 0 \\
 \text{L.H.L. (At } x=0) &= \lim_{h \rightarrow 0} \frac{e^{(0-h)^2} - \cos(0-h)}{(0-h)^2} = \lim_{h \rightarrow 0} \frac{e^{h^2} - \cos(-h)}{h^2} \\
 &= \lim_{h \rightarrow 0} \frac{e^{h^2} - \cos h - 1 + 1}{h^2} = \lim_{h \rightarrow 0} \left(\frac{e^{h^2} - 1}{h^2} + \frac{1 - \cos h}{h^2} \right) \\
 &= \lim_{h \rightarrow 0} \frac{e^{h^2} - 1}{h^2} + \lim_{h \rightarrow 0} \frac{2 \sin^2 \left(\frac{h}{2} \right)}{h^2} \\
 &= 1 + 2 \lim_{h \rightarrow 0} \frac{\sin^2(h/2)}{(h/2)^2} \times \frac{1}{4} = 1 + \frac{2 \times 1}{4} = \frac{3}{2} \\
 \text{Hence, } f(0) &= \frac{3}{2}
 \end{aligned}$$

20. (a): The given LPP can be written as,
Maximize : $z = 2x + y$
subject to $3x + 5y \leq 26$, $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$
Converting the inequations into equations, we obtain the following equations.

$$l_1: 3x + 5y = 26 \text{ or } \frac{x}{\frac{26}{3}} + \frac{y}{\frac{26}{5}} = 1$$

$$l_2: 5x + 3y = 30 \text{ or } \frac{x}{\frac{30}{5}} + \frac{y}{\frac{30}{3}} = 1$$

On solving equations of lines we get the point of intersection $B \left(\frac{9}{2}, \frac{5}{2} \right)$



The shaded region OACB represents the feasible region of the given LPP.

The corner points of the feasible region are O (0, 0), A (6, 0), B $\left(\frac{9}{2}, \frac{5}{2} \right)$ and C $\left(0, \frac{26}{5} \right)$

The values of the objective function at these points are given in the following table.

Point	Value of objective function $z = 2x + y$
O(0,0)	$z = 2 \times 0 + 0 = 0$
A(6,0)	$z = 2 \times 6 + 0 = 12$
B(9/2, 5/2)	$z = 2 \times \frac{9}{2} + \frac{5}{2} = \frac{23}{2} = 11.5$
C(0, 26/5)	$z = 2 \times 0 + \frac{26}{5} = \frac{26}{5} = 5.2$

Clearly maximum value of z is 12 at (6, 0).

21. (c) 22. (d) 23. (a) 24. (a) 25. (b)
26. (a) 27. (c) 28. (c) 29. (c) 30. (b)
31. (b) 32. (c) 33. (a) 34. (d) 35. (c)
36. (c) 37. (b) 38. (d) 39. (a) 40. (b)
41. (a) 42. (b) 43. (a) 44. (d) 45. (a)
46. (d) 47. (d) 48. (a) 49. (c) 50. (c)

For complete solutions refer to
MTG Objective MHT-CET (Mathematics)



MATHS MUSING

SOLUTION SET-186

1. (b) : Let $f(x) = m^2 x^2 + (2m - m^2)x + 3 = 0$... (i)
 $\therefore \alpha, \beta$ are the roots of (i)

$$\therefore \alpha + \beta = \frac{m^2 - 2m}{m^2} = 1 - \frac{2}{m} \text{ and } \alpha\beta = \frac{3}{m^2}.$$

$$\text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2}{\alpha\beta} - 2 = \frac{4}{3}$$

$$\Rightarrow (\alpha + \beta)^2 = \frac{10}{3} \alpha\beta \Rightarrow \left(1 - \frac{2}{m}\right)^2 = \frac{10}{m^2}$$

$$\Rightarrow m^2 - 4m - 6 = 0 \Rightarrow m_1 + m_2 = 4, m_1 m_2 = -6$$

$$\Rightarrow \frac{m_1^2}{m_2} + \frac{m_2^2}{m_1} = \frac{m_1^3 + m_2^3}{m_1 m_2} = \frac{(m_1 + m_2)^3 - 3m_1 m_2 (m_1 + m_2)}{m_1 m_2}$$

$$= \frac{64 + 3 \times 4 \times 6}{-6} = \frac{-68}{3}.$$

2. (b) : $f(x)$ can equivalently be rewritten as:

$$f(x) = \begin{cases} x^2 - 2 & x \leq -1 \\ -x^2 & -1 < x \leq 1 \\ 1 - x & 1 < x < 3/2 \\ 3x - 5 & 3/2 \leq x < 2 \\ x - 1 & 2 \leq x \end{cases}$$

$f(x)$ is non-continuous at $x = 1$

3. (c) : Given, D.E. is of the variable separable form,

$$xy^2 \frac{dy}{dx} = (1 - x^2)(1 + y^2)$$

$$\Rightarrow \left(\frac{y^2}{1 + y^2}\right) dy = \left(\frac{1 - x^2}{x}\right) dx$$

$$\Rightarrow \left(1 - \frac{1}{1 + y^2}\right) dy = \left(\frac{1}{x} - x\right) dx$$

Integrating both sides, we have

$$y - \tan^{-1} y = \ln x - \frac{x^2}{2} + C$$

This is the required general solution.

4. (b) : Let E_i be the event that the student passes the i^{th} examination and E be the event that he qualifies. Thus, E can happen in four possible mutually exclusive ways (sequences):

$$E_1 E_2 \bar{E}_3, E_1 \bar{E}_2 E_3, \bar{E}_1 E_2 E_3, E_1 E_2 E_3.$$

$$\text{Now, } P(E_1 E_2 \bar{E}_3) = p \times p \times (1 - p) = p^2(1 - p)$$

$$P(E_1 \bar{E}_2 E_3) = p \times (1 - p) \times \frac{p}{2} = \frac{p^2}{2}(1 - p)$$

$$P(\bar{E}_1 E_2 E_3) = (1 - p) \times \frac{p}{2} \times p = \frac{p^2}{2}(1 - p)$$

$$P(E_1 E_2 E_3) = p \times p \times p = p^3$$

Thus, $P(E)$ is obtained by adding the four probabilities

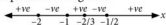
$$\therefore P(E) = 2p^2 - p^3.$$

5. (a) : We have, $\frac{2x}{2x^2 + 5x + 2} - \frac{1}{x + 1} > 0$

$$\Rightarrow \frac{2x(x + 1) - (2x^2 + 5x + 2)}{(x + 1)(2x^2 + 5x + 2)} > 0$$

$$\Rightarrow \frac{-(3x + 2)}{(x + 1)(2x + 1)(x + 2)} > 0 \Rightarrow \frac{(3x + 2)}{(x + 1)(2x + 1)(x + 2)} < 0$$

We mark out the zeroes (of both the numerator and denominator) on a number line and pick the intervals where the expression is negative.



The required values of x are:

$$x \in (-2, -1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$$

6. (c) : Given, $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$

$$\therefore \cos \alpha + \cos \beta = -\cos \gamma \quad \dots (i)$$

$$\sin \alpha + \sin \beta = -\sin \gamma \quad \dots (ii)$$

Squaring and adding (i) and (ii), we get

$$2 + 2\cos(\alpha - \beta) = 1 \Rightarrow \cos(\alpha - \beta) = -\frac{1}{2}$$

$$\text{Similarly, } \cos(\beta - \gamma) = -\frac{1}{2}, \cos(\gamma - \alpha) = -\frac{1}{2}$$

Now, if we square (i) and (ii) and subtract, we get

$$\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta) = \cos 2\gamma$$

$$\Rightarrow 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta) = \cos 2\gamma$$

$$\text{Since, } \cos(\alpha - \beta) = -\frac{1}{2}, \text{ we have}$$

$$\cos(\alpha + \beta) = \cos 2\gamma \quad \dots (iii)$$

and similarly, two other relations.

$$\text{Now, } (\cos \alpha + \cos \beta + \cos \gamma)^2 - (\sin \alpha + \sin \beta + \sin \gamma)^2 = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma + 2[\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha)] = 0$$

Using (iii), this reduces to

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$

$$\text{Since, } \cos 2\alpha = 2\cos^2 \alpha - 1 \text{ and so on.}$$

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2}$$

7. (d) : $G_n = ((n+1)(n+2) \dots (n+n))_n^1$

$$\frac{G_n}{n} = \left(\left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \dots \left(1 + \frac{n}{n}\right) \right)^{\frac{1}{n}}$$

$$\begin{aligned} \therefore \lim_{n \rightarrow \infty} \ln \left(\frac{G_n}{n} \right) &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \ln \left(1 + \frac{r}{n} \right) \\ &= \int_0^1 \ln(1+x) dx = 2 \ln 2 - 1 = \ln \left(\frac{4}{e} \right) \\ \therefore \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f \left(\frac{r}{n} \right) &= \int_0^1 f(x) dx \end{aligned}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{G_n}{n} = \frac{4}{e}$$

$$8. (c) : R_n = \left(\frac{(n+1)^2 + (n+2)^2 + \dots + (n+n)^2}{n} \right)^{\frac{1}{2}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{R_n}{n} \right)^2 = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(1 + \frac{r}{n} \right)^2 = \int_0^1 (1+x)^2 dx = \frac{7}{3}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{R_n}{n} = \sqrt{\frac{7}{3}}$$

$$9. (c) : \text{We have, } \vec{r} \times \vec{b} = \vec{c} \times \vec{b}$$

$$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = \vec{0} \Rightarrow \vec{r} - \vec{c} \text{ is parallel to } \vec{b}$$

$$\text{Thus, } \vec{r} - \vec{c} = \lambda \vec{b} \text{ for some } \lambda \in \mathbb{R}$$

$$\Rightarrow \vec{r} = \vec{c} + \lambda \vec{b} \quad \dots (i)$$

$$\text{Given, } \vec{r} \cdot \vec{a} = 0 \Rightarrow (\vec{c} + \lambda \vec{b}) \cdot \vec{a} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{c} + \lambda \vec{a} \cdot \vec{b} = 0 \Rightarrow \lambda = -\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = -\frac{8+7}{2+0+1} = -5$$

$$\text{Put } \lambda = -5 \text{ in (i) we get}$$

$$\vec{r} = \vec{c} - 5\vec{b} = (4\hat{i} - 3\hat{j} + 7\hat{k}) - 5(\hat{i} + \hat{j} + \hat{k})$$

$$= -\hat{i} - 8\hat{j} + 2\hat{k} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore x + y + z = -7.$$

$$10. (b) : (P) \rightarrow 2, (Q) \rightarrow 1, (R) \rightarrow 2, (S) \rightarrow 3$$

$$(P) \text{ Given, } x^2 + 2x \sin(xy) + 1 = 0$$

$$[x + \sin(xy)]^2 + 1 - \sin^2 xy = 0$$

$$x + \sin(xy) = 0 \text{ \& } \sin^2 xy = 1$$

$$\text{If } \sin^2(xy) = 1 \Rightarrow \sin(xy) = 1 \text{ or } -1$$

$$\text{If } \sin(xy) = 1 \Rightarrow x = -1 \Rightarrow \sin(-y) = 1 \Rightarrow y = 3\pi/2$$

$$\therefore (x, y) = (-1, 3\pi/2)$$

$$\text{If } \sin(xy) = -1 \Rightarrow x = 1 \Rightarrow \sin y = -1 \therefore (x, y) = (1, 3\pi/2)$$

$$(Q) 5 \sin \theta + 3(\sin \theta \cos \alpha - \cos \theta \sin \alpha)$$

$$= (5 + 3 \cos \alpha) \sin \theta - 3 \sin \alpha \cos \theta$$

$$\text{Max. value} = \sqrt{(5 + 3 \cos \alpha)^2 + 9 \sin^2 \alpha} = \sqrt{34 + 30 \cos \alpha}$$

$$\therefore 34 + 30 \cos \alpha = 49 \Rightarrow \cos \alpha = 1/2$$

$$(R) \cos x = |\sin x - \cos x|$$

$$(i) \text{ If } \sin x < \cos x \in \left[0, \frac{\pi}{4} \right) \cup \left(\frac{5\pi}{4}, 2\pi \right]$$

$$\sin x = 0 \Rightarrow x = 2\pi$$

$$(ii) \text{ If } \sin x \geq \cos x \Rightarrow x \in \left[\frac{\pi}{4}, \frac{5\pi}{4} \right]$$

$$\tan x = 2 \Rightarrow x = \tan^{-1} 2$$

$$(S) \sqrt{\sin x} + 2^{1/4} \cos x = 0$$

$$\text{Clearly } \cos x < 0, x \text{ lies } 2^{\text{nd}} \text{ quadrant or } 3^{\text{rd}} \text{ quadrant}$$

$$2^{1/4} \cos x = -\sqrt{\sin x} \Rightarrow \sqrt{2} \cos^2 x = \sin x$$

$$\sin x = \frac{1}{\sqrt{2}}, \therefore x \in 2^{\text{nd}} \text{ quadrant } \therefore x = \frac{3\pi}{4}$$

SAMURAI SUDOKU



Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9×9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3×3 box should contain one of each digit.

The puzzle has a unique solution.

5	1	8							2	9				7	8	1
9							1	8		5	9					4
2							7	9		3	1					5
											3	7				
		6	5	2							6					
	9	3	7								8	1				
8	4						6			8				3	2	
3	7						5			2				8		3
						8	2	7		6	5	3	1			8

						2	6									
						4		3								
						5	1									

	1	8			4	5	9		2		1	8	7			
	4	8					1			6						1
		5		1						8					2	9
			9	4							4		1		9	
				7								4	9	5		
			6	8							3	5	2			
8			6	3						8	6					7
6				1	9					3	9					1
5	7	1			6	8								3	2	6

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Mathdoku Solution Sender

- Mathdoku – Anima Pal (West Bengal)
- Mathdoku – Kaustav Paul
- Sumarai Sudoku Solution – T. I. Darsan

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. Find the value of $f(0)$ such that $f(x)$ becomes continuous at $x = 0$, where

$$f(x) = \frac{\cos^{-1}(1 - \{x\}^2) \sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3}, x \neq 0.$$

Here $\{x\}$ denotes fractional part of x .

(Ankit Jain, M.P.)

Ans. We have,

$$\begin{aligned} \text{R.H.L.} &= \lim_{h \rightarrow 0} f(0-h) \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(1 - \{h\}^2) \sin^{-1}(1 - \{h\})}{\{h\} - \{h\}^3} \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(1 - h^2) \sin^{-1}(1 - h)}{h(1 - h^2)} \\ &= \lim_{h \rightarrow 0} \frac{\sin^{-1}(1 - h) \lim_{h \rightarrow 0} \cos^{-1}(1 - h^2)}{1 - h^2} \\ &= \sin^{-1} 1 \cdot \lim_{\theta \rightarrow 0} \frac{\cos^{-1}(\cos 2\theta)}{\sqrt{2} \sin \theta} \\ &= \frac{\pi}{2\sqrt{2}} \cdot \lim_{\theta \rightarrow 0} \frac{2\theta}{\sin \theta} = \frac{\pi}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{L.H.L.} &= \lim_{h \rightarrow 0} f(0-h) \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(1 - \{-h\}^2) \sin^{-1}(1 - \{-h\})}{\{-h\} - \{-h\}^3} \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(1 - (1-h)^2) \sin^{-1}(1 - (1-h))}{(1-h) - (1-h)^3} \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(h(2-h)) \sin^{-1} h}{(1-h)(2-h)h} \\ &= \lim_{h \rightarrow 0} \frac{\cos^{-1}(h(2-h))}{(1-h)(2-h)} \lim_{h \rightarrow 0} \frac{\sin^{-1} h}{h} = \frac{\cos^{-1} 0}{2} \cdot 1 = \frac{\pi}{4} \end{aligned}$$

Since R.H.L. $\neq$ L.H.L., therefore no value of $f(0)$ can make f continuous at $x = 0$.

2. Find the extreme values of the function $f(x) = [\cos x - 1] + [\cos 2x - 1] + \dots + [\cos nx - 1]$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], n \in \mathbb{I}^+$. (Tina Dabi, H.P.)

Ans. We have

$$\begin{aligned} f(x) &= [\cos x - 1] + [\cos 2x - 1] + \dots + [\cos nx - 1] \\ &= [\cos x] - 1 + [\cos 2x] - 1 + \dots + [\cos nx] - 1 \\ &= -n + [\cos x] + [\cos 2x] + \dots + [\cos nx] \end{aligned}$$

Thus, we have $f_{\max} = -n + n = 0$ at $x = 0$

$$\text{and } f_{\min} = -n + \underbrace{(0-1+0+1+\dots)}_{\text{these values now repeat themselves}} \text{ at } x = \frac{\pi}{2}$$

$$\text{i.e. } f_{\min} = \begin{cases} -n, & \text{if } n = 4k \text{ or } 4k+1 \\ -n-1, & \text{if } n = 4k+2 \text{ or } 4k+3 \end{cases}$$

Hence, the extreme values of $f(x)$ are $-n, -n-1, 0$.

3. In a triangle ABC , D and E are points on BC and AC , respectively, such that $BD = 2DC$ and $AE = 3EC$. Let P be the point of intersection of AD and BE . Find BP/PE using the vector method.

(Maninder Kaur, Punjab)

Ans. Let the vertices of the triangle be $A(\vec{0}), B(\vec{b})$ and $C(\vec{c})$.

Given that D divides BC in the ratio $2 : 1$. Therefore, position

vector of D is $\frac{\vec{b} + 2\vec{c}}{3}$.

E divides AC in the ratio $3 : 1$.

Therefore, position vector of E is $\frac{\vec{0} + 3\vec{c}}{4} = \frac{3\vec{c}}{4}$.

Let point of intersection P of AD and BE divide BE in the ratio $1 : k$ and AD in the ratio $1 : m$. Then position vectors of P in these two cases are

$$\frac{k\vec{b} + 1(3\vec{c}/4)}{k+1} \text{ and } \frac{m\vec{0} + 1(\vec{b} + 2\vec{c}/3)}{m+1}, \text{ respectively.}$$

Equating the position vectors of P in these two cases, we get

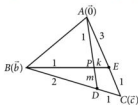
$$\frac{k\vec{b}}{k+1} + \frac{3\vec{c}}{4(k+1)} = \frac{\vec{b}}{3(m+1)} + \frac{2\vec{c}}{3(m+1)}$$

$$\Rightarrow \frac{k}{k+1} = \frac{1}{3(m+1)} \text{ and } \frac{3}{4(k+1)} = \frac{2}{3(m+1)}$$

On dividing above two equations, we get

$$\frac{4k}{3} = \frac{1}{2} \text{ or } k = \frac{3}{8}$$

$\therefore$ Required ratio is $8 : 3$. ◆◆



BRAIN @ WORK



COMPLEX NUMBERS

PROPERTIES OF MODULUS AND ARGUMENT

- $|z_1 + z_2 + \dots + z_n| \leq |z_1| + |z_2| + \dots + |z_n|$
- $|z_1 - z_2| \geq ||z_1| - |z_2||$
- $|z_1 + z_2|^2 = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$
 $= |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + z_2\bar{z}_1$
- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
- $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$

DE MOIVRE'S THEOREM

If $z = r(\cos\theta + i\sin\theta)$ and n is a positive integer, then

$$z^{1/n} = r^{1/n} \left[\cos\left(\frac{2k\pi + \theta}{n}\right) + i\sin\left(\frac{2k\pi + \theta}{n}\right) \right],$$

where, $k = 0, 1, 2, \dots, n-1$

THE n^{th} ROOT OF UNITY

Let x be the n^{th} root of unity.

Then $x = (1)^{1/n} = (\cos 2k\pi + i\sin 2k\pi)^{1/n}$ (where k is an integer)

$$\Rightarrow x = \cos \frac{2k\pi}{n} + i\sin \frac{2k\pi}{n}, \quad k = 0, 1, 2, \dots, n-1$$

$$\text{Let } \alpha = \cos \frac{2\pi}{n} + i\sin \frac{2\pi}{n}$$

Then the n^{th} roots of unity are α^t ($t = 0, 1, 2, \dots, n-1$), i.e. the n^{th} roots of unity are $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$.

Sum of the Roots

$$1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} = \frac{1 - \alpha^n}{1 - \alpha} = 0$$

$$\Rightarrow \sum_{k=0}^{n-1} \cos \frac{2k\pi}{n} = 0 \quad \text{and} \quad \sum_{k=0}^{n-1} \sin \frac{2k\pi}{n} = 0$$

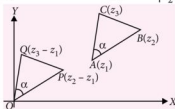
Thus the sum of the roots of unity is zero.

Product of the Roots

$$1 \cdot \alpha \cdot \alpha^2 \cdot \dots \cdot \alpha^{n-1} = (-1)^{n-1}(-1) = (-1)^{n-1}$$

Concept of Rotation : If z_1, z_2, z_3 are the three vertices of a triangle ABC described in the counter-clock wise sense, then

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{OQ}{OP} (\cos \alpha + i\sin \alpha) = \frac{CA}{BA} \cdot e^{i\alpha} = \frac{|z_3 - z_1|}{|z_2 - z_1|} \cdot e^{i\alpha}$$



GEOMETRICAL APPLICATIONS

- **Condition for Collinearity :** If there are three real numbers (other than 0) l, m and n such that $lz_1 + mz_2 + nz_3 = 0$ and $l + m + n = 0$ then complex numbers z_1, z_2 and z_3 will be collinear.

Equation of a Straight Line

- **Equation of a straight line with the help of rotation formula:**

Let $A(z_1)$ and $B(z_2)$ be any two points lying on any line and we have to obtain the equation of this line. For this purpose let us take any point $C(z)$ on this line.

$$\text{since } \arg \left(\frac{z - z_1}{z_2 - z_1} \right) = 0 \text{ or } \pi \Rightarrow \frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \dots (i)$$

- **General equation of the line:**

The general equation of a line in the complex plane is. $z\bar{a} + \bar{z}a + b = 0$

where, a is non zero complex number and b is any real number

- **Slope of a given line:**

If $z\bar{a} + \bar{z}a + b = 0$ is the given line then its slope $= -\frac{a}{\bar{a}}$

- Equation of a line parallel to the line $z\bar{a} + \bar{z}a + b = 0$ is $z\bar{a} + \bar{z}a + \lambda = 0$ (where λ is a real number).

- Equation of a line perpendicular to the line $z\bar{a} + \bar{z}a + b = 0$ is $z\bar{a} - \bar{z}a + i\lambda = 0$. (where λ is a real number).

Equation of Perpendicular Bisector:

Consider a line segment joining $A(z_1)$ and $B(z_2)$. Let the line 'L' be its perpendicular bisector. If $P(z)$ be any point on the line L, we have $PA = PB$.

$$\Rightarrow |z - z_1| = |z - z_2|$$

$$\Rightarrow z(\bar{z}_2 - \bar{z}_1) + \bar{z}(z_2 - z_1) + z_1\bar{z}_1 - z_2\bar{z}_2 = 0$$

- Distance of a given point from a given line:**

Let the given line be $z\bar{a} + \bar{z}a + b = 0$, and the given point be z_c then distance of z_c from this line is $\frac{|z_c\bar{a} + \bar{z}_ca + b|}{2|a|}$.

Equation of a Circle

Equation of a circle of radius r and having its centre at z_0 is $|z - z_0| = r$.

$$\Rightarrow |z - z_0|^2 = r^2 \Rightarrow (z - z_0)(\bar{z} - \bar{z}_0) = r^2$$

$$\Rightarrow z\bar{z} + a\bar{z} + \bar{a}z + b = 0$$

where $-a = z_0$ and $b = z_0\bar{z}_0 - r^2$

It represents the general equation of a circle in the complex plane.

- Equation of a circle described on a line segment $AB, (A(z_1), B(z_2))$ as diameter is $(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0$

- Let z_1 and z_2 be two given complex numbers and z be a any complex number such that,

$\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$, where $\alpha \in (0, \pi)$. Then 'z' will lie on the arc of a circle.

- Let $ABCD$ be a cyclic quadrilateral such that $A(z_1), B(z_2), C(z_3)$ and $D(z_4)$ lie on a circle. Then $\frac{(z_4 - z_1)(z_3 - z_2)}{(z_4 - z_2)(z_3 - z_1)}$ is purely real.

Some Important Result to Remember

- The triangle whose vertices are the points represented by complex numbers z_1, z_2, z_3 is equilateral,

$$\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0$$

$$\Leftrightarrow z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1.$$

- $|z - z_1| + |z - z_2| = \lambda$, represents an ellipse, if $|z_1 - z_2| < \lambda$, having the points z_1 and z_2 as its foci. And if $|z_1 - z_2| = \lambda$, then z lies on a line segment connecting z_1 and z_2 .

- $|z - z_1| - |z - z_2| = \lambda$, represents a hyperbola, if $|z_1 - z_2| \neq \lambda$, having the points z_1 and z_2 as its foci. And if $|z_1 - z_2| = \lambda$, then z lies on the line passing through z_1 and z_2 excluding the points between z_1 and z_2 .

PROBLEMS

- Number of solutions of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$, where z is a complex number is

(a) 2 (b) 3 (c) 6 (d) 5

- Let z_1 and z_2 be n^{th} roots of unity. If origin, z_1 and z_2 form an equilateral triangle. Then n must be of the form (a) $6k$ (b) $6k + 1$ (c) $6k + 2$ (d) $6k + 5$

- If $|z| = r$ and area of the triangle whose vertices are $z, \omega z$ and $z + \omega z$, (where ω is the non real cube root of unity), is $4\sqrt{3}$ sq. units, then the value of ' r ' is (a) 1 unit (b) 2 units (c) 3 units (d) 4 units

- If $|z| \leq 4$, then the maximum value of $|iz + 3 - 4i|$ is equal to

(a) 2 (b) 4 (c) 3 (d) 9

- If p and q are distinct prime numbers, then the number of distinct imaginary numbers which are p^{th} as well as q^{th} roots of unity are

(a) $\min(p, q)$ (b) $\max(p, q)$
(c) 1 (d) zero

- If z_1, z_2, z_3 are three distinct complex numbers

such that $\frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|}$, then the value

of $\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2}$ equals

(a) 0 (b) 3 (c) 4 (d) 5

- Let z is complex satisfying the equation $z^2 - (3 + i)z + m + 2i = 0$, where $m \in \mathbb{R}$. Suppose the equation has a real root, then the additive inverse of non real root, is

(a) $1 - i$ (b) $1 + i$ (c) $-1 - i$ (d) -2

- The principal argument of the complex number $\frac{(1 + i)^5(1 + \sqrt{3}i)^2}{-2i(-\sqrt{3} + i)}$ is

(a) $\frac{19\pi}{12}$ (b) $-\frac{7\pi}{12}$ (c) $-\frac{5\pi}{12}$ (d) $\frac{5\pi}{12}$

- Let $z = x + iy$, then locus of moving point $P(z)$ such that $\frac{1 + \bar{z}}{z} \in \mathbb{R}$, (where $i^2 = -1$) is

- (a) union of lines with equations $x = 0$ and $y = -1/2$ but excluding origin.
 (b) union of lines with equations $x = 0$ and $y = 1/2$ but excluding origin.
 (c) union of lines with equations $x = -1/2$ and $y = 0$ but excluding origin.
 (d) union of lines with equations $x = 1/2$ and $y = 0$ but excluding origin.

10. If $|z| = 1$ and $|\omega - 1| = 1$ where $z, \omega \in \mathbb{C}$, then the largest set of values of $|2z - 1|^2 + |2\omega - 1|^2$ equals
 (a) $[1, 9]$ (b) $[2, 6]$ (c) $[2, 12]$ (d) $[2, 18]$

11. Let z be a complex number having the argument θ , ($0 < \theta < \pi/2$) and satisfying the equality $|z - 3i|$. Then $\cot \theta - \frac{6}{z}$ is equal to

- (a) 1 (b) -1 (c) i (d) $-i$

12. If $1, \alpha_1, \alpha_2, \dots, \alpha_{2008}$ are $(2009)^{\text{th}}$ roots of unity, then the value of $\sum_{r=1}^{2008} r(\alpha_r + \alpha_{2009-r})$ equals

- (a) 2009 (b) 2008 (c) 0 (d) -2009

13. If z and w are complex numbers satisfying $\bar{z} + i\bar{w} = 0$ and $\text{amp}(zw) = \pi$, then $\text{amp}(z)$ is equal to
 (a) $\pi/4$ (b) $-\pi/2$ (c) $\pi/2$ (d) $3\pi/4$

14. If the equation, $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$, where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root, then the expression

$$\frac{a_3}{a_1 a_2} + \frac{a_1 a_4}{a_2 a_3}$$

- (a) 0 (b) 1 (c) -2 (d) 2

15. If z_r ; $r = 1, 2, 3, \dots, 50$ are the roots of the equation $\sum_{r=0}^{50} (z)^r = 0$, then the value of $\sum_{r=1}^{50} \frac{1}{z_r - 1}$ is

- (a) -85 (b) -25 (c) 25 (d) 75

16. The minimum value of $|z - 1 + 2i| + |4i - 3 - z|$ is
 (a) $\sqrt{5}$ (b) 5 (c) $2\sqrt{13}$ (d) $\sqrt{15}$

17. If the complex numbers x and y satisfy $x^3 - y^3 = 98i$ and $x - y = 7i$, then $xy = a + ib$ where $a, b \in \mathbb{R}$. The value of $(a + b)$ equals

- (a) 21 (b) 18
 (c) -18 (d) None of these

18. The complex number z having least positive argument which satisfy the condition $|z - 25i| \leq 15$ is
 (a) $25i$ (b) $12 + 25i$ (c) $16 + 12i$ (d) $12 + 16i$

19. All complex numbers z which satisfy the relation $|z - |z + 1|| = |z + |z - 1||$ on the complex plane lie on the

- (a) line $y = 0$ or an ellipse with foci $(-1, 0)$ and $(1, 0)$.
 (b) radical axis of the circles $|z - 1| = 1$ and $|z + 1| = 1$.
 (c) circle $x^2 + y^2 = 1$.
 (d) line $x = 0$ or on a line segment joining $(-1, 0)$ to $(1, 0)$.

20. If z is a complex number satisfying the equation $|z - (1 + i)|^2 = 2$ and $\omega = 2/z$, then the locus traced by ' ω ' in the complex plane is

- (a) $x - y - 1 = 0$ (b) $x + y - 1 = 0$
 (c) $x - y + 1 = 0$ (d) $x + y + 1 = 0$

21. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 2|$, $\text{Re}(z_2) = |z_2 - 2|$ and $\arg(z_1 - z_2) = \frac{\pi}{3}$, then $\text{Im}(z_1 + z_2) =$

- (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) $\sqrt{3}$

22. If z be a complex number such that $|z - \alpha^2| + |z - 4\alpha| = 5$ always represents an ellipse, then range of $\alpha (\in \mathbb{R}^+)$ is

- (a) $(0, 5)$ (b) $(1, 6)$ (c) $(2, 6)$ (d) $(3, 7)$

23. If $\alpha = -1 + i\sqrt{3}$ and n is a positive integer which is not a multiple of 3, then $\alpha^{2n} + 2^n \alpha^n + 2^{2n} =$
 (a) 1 (b) -1 (c) 0 (d) α^2

24. Let $\omega \neq 1$ be a cube root of unity and $E = 2(1 + \omega)(1 + \omega^2) + 3(2\omega + 1)(2\omega^2 + 1) + 4(3\omega + 1)(3\omega^2 + 1) \dots + (n + 1)(n\omega + 1)(n\omega^2 + 1)$. Then E is equal to

- (a) $\frac{n^2(n+1)^2}{4}$ (b) $\frac{n^2(n+1)^2}{4} + n$
 (c) $\frac{n^2(n+1)^2}{4} - n$ (d) $\frac{n^2(n+1)^2}{4} - (n+1)$

25. A particle starts from a point $z_0 = 1 + i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 2 units and then vertically away from origin by 3 units to reach a point z_1 . From z_1 , particle moves $\sqrt{5}$ units in the direction of $2\hat{i} + \hat{j}$ and then it moves through an angle of $\text{cosec}^{-1}\sqrt{2}$ in anticlockwise direction of a circle with centre at origin to reach a point z_2 . The arg z_2 is given by

- (a) $\sec^{-1}2$ (b) $\cot^{-1}0$
 (c) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ (d) $\cos^{-1}\left(\frac{-1}{2}\right)$

26. The continued product of all the four values of the complex number $(1 + i)^{3/4}$ is

- (a) $2^3(1 + i)$ (b) $2(1 - i)$ (c) $2(1 + i)$ (d) $2^3(1 - i)$

27. Locus of z satisfying the equation $|z - 4| + |z + 4| = 16$ is conic C , then equation of conic which is confocal with C (have the same focus) and whose eccentricity is reciprocal of the eccentricity of conic C , is given by

- (a) $|z - 4| - |z + 4| = 4$ (b) $|z - 4| + |z + 4| = 8$
(c) $|z - 4| - |z + 4| = 8$ (d) $|z - 4| + |z + 4| = 4$

28. Let $z = \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{i}{2}\right)$. Then the smallest positive

integer n such that $(z^{95} + i^{67})^{94} = z^n$ is

- (a) 12 (b) 10 (c) 9 (d) 8

29. If z_1, z_2 are the roots of the quadratic equation $az^2 + bz + c = 0$ such that $\text{Im}(z_1, z_2) \neq 0$, then

- (a) a, b, c are all real
(b) at least one of a, b, c is real
(c) at least one of a, b, c is imaginary
(d) all of a, b, c are imaginary

30. If $1, \alpha_1, \alpha_2, \alpha_3, \dots$ and α_8 are nine, ninth roots of unity (taken in counter-clockwise sequence) then $|(2 - \alpha_1)(2 - \alpha_3)(2 - \alpha_5)(2 - \alpha_7)|$ is equal to

- (a) $\sqrt{255}$ (b) $\sqrt{511}$ (c) $\sqrt{1023}$ (d) 15

31. The point of intersection the curves $\arg(z - i + 2) = \frac{\pi}{6}$ and $\arg(z + 4 - 3i) = -\frac{\pi}{4}$ is given by

- (a) $-2 + i$ (b) $2 - i$
(c) $2 + i$ (d) none of these

32. If $|z_2 + iz_1| = |z_1| + |z_2|$ and $|z_1| = 3$ and $|z_2| = 4$ then area of ΔABC , if affix of A, B and C are $(z_1), (z_2)$ and $\left(\frac{z_2 - iz_1}{1 - i}\right)$ respectively (in sq. units), is

- (a) $\frac{5}{2}$ (b) 0 (c) $\frac{25}{2}$ (d) $\frac{25}{4}$

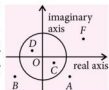
33. There is only one way to choose real numbers M and N such that when the polynomial $5x^4 + 4x^3 + 3x^2 + Mx + N$ is divided by the polynomial $x^2 + 1$, the remainder is 0. If M and N assume these unique values, then $M - N$ is

- (a) -6 (b) -2 (c) 6 (d) 2

34. Let Z be a complex number satisfying the equation $(Z^3 + 3)^2 = -16$, then $|Z|$ has the value equal to

- (a) $5^{1/2}$ (b) $5^{1/3}$ (c) $5^{2/3}$ (d) 5

35. The diagram shows several numbers in the complex plane. The circle is the unit circle centered at the origin. One of these numbers is the reciprocal of F , which is



- (a) A (b) B (c) C (d) D

36. If t and c are two complex numbers such that

$|t| \neq |c|, |t| = 1$ and $z = \frac{at + b}{t - c}, z = x + iy$. Locus of z is (where a, b are complex numbers)

- (a) line segment (b) straight line
(c) circle (d) none

37. Consider the equation $10z^2 - 3iz - k = 0$, where z is a complex variable and $i^2 = -1$. Which of the following statement(s) is(are) true?

- (a) For all real positive numbers k , both roots are pure imaginary.
(b) For negative real numbers k , both roots are pure imaginary.
(c) For all pure imaginary numbers k , both roots are real and irrational.
(d) For all complex numbers k , neither root is real.

38. If $\text{Arg}(z + a) = \pi/6$ and $\text{Arg}(z - a) = 2\pi/3$, then $a \in \mathbb{R}^+$

- (a) z is independent of a (b) $|a| = |z + a|$
(c) $\arg(z) = \pi/6$ (d) $\arg(z) = \pi/3$

39. A root of unity is a complex number that is a solution to the equation, $z^n = 1$ for some positive integer n . Number of roots of unity that are also the roots of the equation $z^2 + az + b = 0$, for some integer a and b is

- (a) 6 (b) 8 (c) 9 (d) 10

40. If z is a complex number satisfying the equation $z^6 + z^3 + 1 = 0$. If this equation has a root $re^{i\theta}$ with $90^\circ < \theta < 180^\circ$, then the value of θ is

- (a) 100° (b) 110° (c) 160° (d) 170°

SOLUTIONS

1. (d): Since, $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$.

Let $z = re^{iq} \Rightarrow r^3 e^{i3q} + 3r e^{-i2q} = 0$

Since r cannot be zero.

$\Rightarrow r^2 e^{i5q} = -3$ which will hold for $r = \sqrt{3}$ and 5 distinct values of q .

Thus, there are five solution.

2. (a): $\therefore n^{\text{th}}$ root is of the form

$\cos\left(\frac{2r\pi}{n}\right) + i\sin\left(\frac{2r\pi}{n}\right)$, where $r = 0, 1, 2, \dots, n - 1$

Let $z_1 = \cos\left(\frac{2r_1\pi}{n}\right) + i\sin\left(\frac{2r_1\pi}{n}\right)$

and $z_2 = \cos\left(\frac{2r_2\pi}{n}\right) + i\sin\left(\frac{2r_2\pi}{n}\right)$,

$r_1, r_2 \in [0, 1, 2, \dots, n - 1]$

Since, origin, z_1 and z_2 form an equilateral triangle.

$$\therefore \frac{2(r_1 - r_2)\pi}{n} = \frac{\pi}{3} \Rightarrow n = 6(r_1 - r_2)$$

$\therefore n$ must be of the form $6k$.

3. (d) : Triangle formed by the points $A(z)$, $B(\omega z)$ and

$C(z + \omega z)$ is $\frac{\sqrt{3}}{4} |z|^2$ sq. units.

$$\Rightarrow \text{Ar}(\triangle ABC) = r^2 \frac{\sqrt{3}}{4} = 4\sqrt{3} \Rightarrow r = 4 \text{ units}$$

$$4. (d) : |iz + 3 - 4i| \leq |iz| + |3 - 4i| = |z| + 5 \leq 9$$

$$5. (d) : \because e^{\frac{2\pi r_i}{p}} = e^{\frac{2\pi m_j}{q}}$$

where $r = 0, 1, \dots, p-1$ and $m = 0, 1, \dots, q-1$

This is possible iff $r = m = 0$

But for $r = m = 0$, we get 1 which is not an imaginary number.

$$6. (a) : \text{Let } \frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|} = k$$

$$\Rightarrow \frac{9}{|z_2 - z_3|^2} = \frac{16}{|z_3 - z_1|^2} = \frac{25}{|z_1 - z_2|^2} = k^2$$

$$\text{Now, } \frac{9}{|z_2 - z_3|^2} = k^2 \Rightarrow \frac{9}{z_2 - z_3} = k^2(\bar{z}_2 - \bar{z}_3) \quad [A_s, |z|^2 = z\bar{z}]$$

$$\frac{16}{|z_3 - z_1|^2} = k^2 \Rightarrow \frac{16}{z_3 - z_1} = k^2(\bar{z}_3 - \bar{z}_1) \quad \dots(2)$$

$$\frac{25}{|z_1 - z_2|^2} = k^2 \Rightarrow \frac{25}{z_1 - z_2} = k^2(\bar{z}_1 - \bar{z}_2) \quad \dots(3)$$

On adding (1), (2) and (3), we get

$$\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2} = 0$$

7. (c) : Let α be the real root of the given equation.

$$\text{Then, } \alpha^2 - (3+i)\alpha + m + 2i = 0$$

$$\Rightarrow (\alpha^2 - 3\alpha + m) + i(2 - \alpha) = 0$$

$$\therefore \alpha = 2 \text{ (real root)} \quad \therefore 4 - 6 + m = 0 \Rightarrow m = 2$$

Product of the roots = $2(1+i)$ with one root as 2

and non real root is $1+i$, additive inverse is $-1-i$.

$$8. (c) : \arg \left(\frac{(1+i)^5(1+\sqrt{3}i)^2}{-2i(-\sqrt{3}+i)} \right) \\ = \arg \left(\frac{(\sqrt{2})^5 \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)^5 \cdot 2^2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)^2}{2i \times 2 \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)} \right)$$

$$\therefore \frac{5\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{2} + \frac{\pi}{6} = \frac{19\pi}{12}$$

$$\therefore \text{Principal argument is } -\frac{5\pi}{12}$$

$$9. (c) : \text{Given } \frac{1+\bar{z}}{z} \text{ is real} \Rightarrow \frac{1+\bar{z}}{z} = \frac{1+z}{\bar{z}}$$

$$\Rightarrow \bar{z} + \bar{z}^2 = z + z^2$$

$$\Rightarrow (\bar{z} - z) + (\bar{z} - z)(\bar{z} + z) = 0 \Rightarrow (\bar{z} - z)(1 + \bar{z} + z) = 0$$

So either $\bar{z} = z$ ($z \neq 0$) or $z + \bar{z} + 1 = 0$

$$\Rightarrow y = 0 \text{ or } x = -\frac{1}{2} \text{ but excluding origin.}$$

$$10. (d) : \text{Let } E = (2z - 1)(2\bar{z} - 1) + (2\omega - 1)(2\bar{\omega} - 1)$$

$$= [4|z|^2 - 2(z + \bar{z}) + 1] + [4|\omega|^2 - 2(\omega + \bar{\omega}) + 1]$$

$$= 6 - 4\text{Re}(z) + 4\omega\bar{\omega} - 4\text{Re}(\omega)$$

$$\text{Now } |\omega - 1|^2 = 1 \Rightarrow (\omega - 1)(\bar{\omega} - 1) = 1$$

$$\omega\bar{\omega} - (\omega + \bar{\omega}) = 0 \Rightarrow \omega\bar{\omega} = 2\text{Re}(\omega)$$

$$\text{Hence } E = 6 - 4\text{Re}(z) + 4\text{Re}(\omega) = 6 + 4(x_2 - x_1)$$

$$\therefore E_{\max} = 6 + 4(2 + 1) = 18$$

$$E_{\min} = 6 + 4(0 - 1) = 2$$

$\therefore$ Required set of values is $[2, 18]$

$$11. (c) : \text{Let } z = r(\cos\theta + i\sin\theta)$$

$$\text{Now, } r = OA \sin\theta = 6 \sin\theta$$

$$\therefore z = 6 \sin\theta (\cos\theta + i\sin\theta)$$

$$\Rightarrow \frac{6}{z} = \frac{1}{\sin(\cos\theta + i\sin\theta)} \\ = \frac{\cos\theta - i\sin\theta}{\sin\theta} = -i + \cot\theta$$

$$\Rightarrow \cot\theta - \frac{6}{z} = i$$

$$12. (d) : \text{Let } S = 1(\alpha_1 + \alpha_{2008}) + 2(\alpha_2 + \alpha_{2007})$$

$$+ 3(\alpha_3 + \alpha_{2006}) + \dots + 2008(\alpha_{2008} + \alpha_1) \quad \dots(1)$$

$$\text{Also, } S = 2008(\alpha_{2008} + \alpha_1) + 2007(\alpha_2 + \alpha_{2007}) +$$

$$\dots + 2(\alpha_2 + \alpha_{2007}) + 1(\alpha_1 + \alpha_{2008}) \quad \dots(2)$$

(writing in reverse order)

$\therefore$ On adding (1) and (2), we get

$$2S = 2009[2(\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_{2008})]$$

$$\text{or } 2S = 2009[2(\underbrace{1 + \alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_{2008}}_{\text{zero}} - 1)]$$

$$\text{Hence, } S = -2009$$

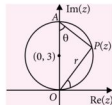
$$13. (d) : \text{Given, } \bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w} \text{ or } z = iw$$

$$\text{amp}(z) - \text{amp}(w) = \text{amp}(i) = \pi/2 \quad \dots(1)$$

$$\text{Also, amp}(zw) = \pi \Rightarrow \text{amp}(z) + \text{amp}(w) = \pi \quad \dots(2)$$

$$\text{Adding (1) and (2), we get } 2\text{amp}(z) = \frac{3\pi}{2}$$

$$\Rightarrow \text{amp}(z) = \frac{3\pi}{4}$$



14. (b): Let x_i be the root where $x \neq 0$ and $x \in \mathbb{R}$ (as if $x = 0$ satisfies, then $a_4 = 0$ which contradicts)

We have, $x^4 - a_1x^3 - a_2x^2 + a_3xi + a_4 = 0$

$$\Rightarrow x^4 - a_2x^2 + a_4 = 0 \quad \dots(1)$$

$$\text{and } a_1x^3 - a_3x = 0 \quad \dots(2)$$

$$\text{From (2), } a_1x^2 - a_3 = 0 \Rightarrow x^2 = a_3/a_1$$

Putting the value of x^2 in equation (1)

$$\frac{a_3^2}{a_1^2} - \frac{a_2a_3}{a_1} + a_4 = 0 \quad \text{or} \quad \frac{a_3}{a_1a_2} + \frac{a_1a_4}{a_2a_3} = 1$$

15. (b): Let $1 + z + z^2 + \dots + z^{50} = (z - z_1)(z - z_2)(z - z_{50})$

Differentiating both sides w.r.t. z after taking logarithm on both the sides, we get

$$\frac{1 + 2z + 3z^2 + \dots + 50z^{49}}{1 + z + z^2 + \dots + z^{50}} = \frac{1}{z - z_1} + \frac{1}{z - z_2} + \dots + \frac{1}{z - z_{50}}$$

Now put $z = 1$ we get,

$$\frac{50 \cdot 51}{2 \cdot 51} = - \left[\frac{1}{z_1 - 1} + \frac{1}{z_2 - 1} + \dots + \frac{1}{z_{50} - 1} \right]$$

$$\therefore \sum_{r=1}^{50} \frac{1}{z_r - 1} = -25$$

16. (c): The expression is the sum of the distance of z from the two points $1 - 2i$ and $-3 + 4i$. The minimum value is the distance between these two points

$$= \sqrt{4^2 + 6^2} = 2\sqrt{13}$$

17. (a): $x^3 - y^3 = 98i$

$$\Rightarrow (x - y)^3 + 3xy(x - y) = 98i$$

$$\Rightarrow -343i + 3(a + ib)(7i) = 98i$$

$$\Rightarrow -343 + 3(a + bi)7i = 98 \Rightarrow -49 + 3(a + bi) = 14$$

$$\Rightarrow 3(a + bi) = 63 \Rightarrow a + ib = 21$$

$$\Rightarrow a = 21 \text{ and } b = 0 \Rightarrow a + b = 21$$

18. (d): The required complex number is point of contact P as shown. $C(0, 25)$ is the centre of the circle and radius is 15.

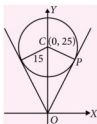
$$\text{Now } |z| = OP = \sqrt{OC^2 - PC^2} = \sqrt{625 - 225} = 20$$

$$\text{amp}(z) = \theta = \angle XOP = \angle OCP$$

$$\therefore \cos \theta = \frac{PC}{OC} = \frac{15}{25} = \frac{3}{5}$$

$$\text{and } \sin \theta = \frac{OP}{OC} = \frac{20}{25} = \frac{4}{5}$$

$$\therefore z = 20 \left(\frac{3}{5} + \frac{4}{5}i \right) = 12 + 16i$$



19. (d)

20. (a): We have $|z - (1 + i)|^2 = 2$

$$\Rightarrow (x - 1)^2 + (y - 1)^2 = 2 \quad (\text{Put } z = x + iy)$$

$$\Rightarrow x^2 + y^2 = 2(x + y) \quad \dots (1)$$

$$\text{Let } \omega = h + ik = \frac{2}{z} = \frac{2}{x + iy} = \frac{2(x - iy)}{x^2 + y^2}$$

$$\text{So, } h = \frac{2x}{x^2 + y^2}, k = \frac{-2y}{x^2 + y^2}$$

$$\Rightarrow h - k = \frac{2(x + y)}{x^2 + y^2} = 1 \quad (\text{from equation (1)})$$

$\therefore$ Locus of the point $\omega(h, k)$ will be $x - y = 1$

21. (b): Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Then $x_1^2 = (x_1 - 2)^2 + y_1^2$ and $x_2^2 = (x_2 - 2)^2 + y_2^2$

Therefore

$$4x_1 = y_1^2 + 4 \text{ and } 4x_2 = y_2^2 + 4$$

$$\Rightarrow 4(x_1 - x_2) = y_1^2 - y_2^2 = (y_1 + y_2)(y_1 - y_2)$$

$$\text{Hence, } y_1 + y_2 = \frac{4(x_1 - x_2)}{y_1 - y_2} \quad \dots(1)$$

$$\text{Also, } \arg(z_1 - z_2) = \frac{\pi}{3}$$

$$\tan \frac{\pi}{3} = \frac{y_1 - y_2}{x_1 - x_2} \Rightarrow \sqrt{3} = \frac{y_1 - y_2}{x_1 - x_2} \quad \dots(2)$$

$\therefore$ From (1) and (2), we have,

$$\text{Im}(z_1 + z_2) = y_1 + y_2 = \frac{4}{\sqrt{3}}$$

22. (a): $|\alpha^2 - 4\alpha| < 5$

$$\Rightarrow -5 < \alpha^2 - 4\alpha < 5 \Rightarrow -1 < (\alpha - 2)^2 < 9$$

$$\Rightarrow 0 \leq (\alpha - 2)^2 < 9 \Rightarrow -3 < \alpha - 2 < 3$$

$$\Rightarrow -1 < \alpha < 5 \Rightarrow 0 < \alpha < 5$$

23. (c): We have,

$$\alpha^{2n} + 2^n \alpha^n + 2^{2n} = 2^{2n} \left[\left(\frac{\alpha}{2} \right)^{2n} + \left(\frac{\alpha}{2} \right)^n + 1 \right]$$

$$= 2^{2n} (\omega^{2n} + \omega^n + 1) \left[\because \frac{\alpha}{2} = \frac{-1 + i\sqrt{3}}{2} \text{ and } \left(\frac{\alpha}{2} \right)^3 = 1 \right]$$

$$= 2^{2n}(0) = 0 \quad (\text{Since 3 does not divide } n)$$

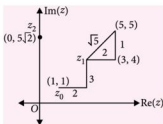
24. (b): We have $(k + 1)(k\omega + 1)(k\omega^2 + 1) = (k + 1)(k^2 - k + 1) = k^3 + 1$

$$\text{Therefore } E = \sum_{k=1}^n (k^3 + 1) = \sum_{k=1}^n k^3 + n = \frac{n^2(n+1)^2}{4} + n$$

25. (b): Clearly $z_1 = 3 + 4i$

After moving by distance in direction of $2\hat{i} + \hat{j}$, particle will reach at point $(5\hat{i} + 5\hat{j})$

If particle moves by an angle $\frac{\pi}{4}$, then it will reach at y-axis.



At $z_2 = 0 + 5\sqrt{2}i$

Hence, $\arg(z_2) = \frac{\pi}{2} = \cot^{-1}0$

26. (b) : Let $z = 1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$

There fore $z^{3/4} = 2^{3/8} \left[\cos \left(2k\pi + \frac{\pi}{4} \right)^{3/4} + i \sin \left(2k\pi + \frac{\pi}{4} \right)^{3/4} \right]$

for $k = 0, 1, 2, 3$.

Required product = $2^{3/2} \left[\cos \left(\frac{\pi}{4} + \frac{9\pi}{4} + \frac{17\pi}{4} + \frac{25\pi}{4} \right)^{3/4} \right]$

$= 2^{3/2} \cos \left(\frac{52\pi}{4} \times \frac{3}{4} \right) = 2^{3/2} \cos \frac{39\pi}{4}$

$= 2^{3/2} \cos \left(9\pi + \frac{3\pi}{4} \right) = 2^{3/2} \cos \left(10\pi - \frac{\pi}{4} \right)$

$= 2^{3/2} \left[\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right] = 2(1 - i)$

27. (a) : Clearly, locus of $|z - 4| + |z + 4| = 16$ is ellipse with foci (4, 0) and (-4, 0) and eccentricity = $\frac{1}{2}$.

A conic with foci (4, 0) and (-4, 0) and eccentricity 2 will be a hyperbola and its equation will be $|z - 4| - |z + 4| = 4$.

28. (b) : From the hypothesis, we have

$z = \frac{\sqrt{3}}{2} - \frac{i}{2} = i \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2} \right) = i\omega$

where $\omega = \left(-\frac{1}{2} \right) - \left(\frac{i\sqrt{3}}{2} \right)$ which is a cube root of unity.

Now $z^{95} = (i\omega)^{95} = -i\omega^2$ (since $\omega^3 = 1$)

and $i^{67} = i^3 = -i$.

Therefore, $z^{95} + i^{67} = -i(1 + \omega^2) = (-i)(-\omega) = i\omega$

$\Rightarrow (z^{95} + i^{67})^{94} = (i\omega)^{94} = i^2\omega = -\omega$

Now $-\omega = z^n = (i\omega)^n \Rightarrow i^n \cdot \omega^{n-1} = -1$

$\Rightarrow n = 2, 6, 10, 14, \dots$ and $n - 1 = 3, 5, 9, \dots$

Therefore $n = 10$ is the required least positive integer.

29. (c) : $az^2 + bz + c = 0 \dots (1)$

and z_1, z_2 (roots of (1)) are such that $\text{Im}(z_1, z_2) \neq 0$

$\Rightarrow z_1$ and z_2 are not conjugates of each other.

$\Rightarrow$ Complex roots of (1) are not conjugate of each other.

$\Rightarrow$ Coefficient a, b, c can not all be real.

$\Rightarrow$ At least one of a, b, c is imaginary.

30. (b) : $(x - 1)(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_8) \equiv x^9 - 1$

$\therefore (2 - \alpha_1)(2 - \alpha_2) \dots (2 - \alpha_8) = 2^9 - 1$

Now since $(2 - \alpha_1)$ and $(2 - \alpha_8)$ are conjugates of each other.

$\therefore |2 - \alpha_1| = |2 - \alpha_8|$

Similarly, $|2 - \alpha_2| = |2 - \alpha_7|$, $|2 - \alpha_3| = |2 - \alpha_6|$

and $|2 - \alpha_4| = |2 - \alpha_5|$

$\therefore |(2 - \alpha_1)(2 - \alpha_3)(2 - \alpha_5)(2 - \alpha_7)|$

$= \sqrt{2^9 - 1} = \sqrt{511}$

31. (d) : $\arg(z - i + 2) = \frac{\pi}{6} \Rightarrow \tan \frac{\pi}{6} = \frac{y - 1}{x + 2}$

$\Rightarrow x - \sqrt{3}y = -(\sqrt{3} + 2)$, $x > -2$, $y > 1 \dots (1)$

Now, $\arg(z + 4 - 3i) = -\frac{\pi}{4} \Rightarrow \tan \left(-\frac{\pi}{4} \right) = \frac{y - 3}{x + 4}$

$\Rightarrow y + x = -1$, $x > -4$, $y < 3 \dots (2)$

So, there is no point of intersection.

32. (d) : $|z_2 + iz_1| = |z_1| + |z_2|$

$\Rightarrow \arg(iz_1) = \arg(z_2)$

$\Rightarrow \arg(z_2) - \arg(z_1) = \frac{\pi}{2}$

Let $z_3 = \frac{z_2 - iz_1}{1 - i}$

$\Rightarrow (1 - i)z_3 = z_2 - iz_1$

$\Rightarrow (z_3 - z_2) = i(z_3 - z_1) \Rightarrow (z_2 - z_3) = i(z_1 - z_3)$

$\therefore \angle ACB = \frac{\pi}{2}$ and $|z_2 - z_3| = |z_1 - z_3| \Rightarrow AC = BC$

$AB^2 = AC^2 + BC^2 \Rightarrow AC = \frac{5}{\sqrt{2}} (\because AB = 5)$

$\therefore \text{ar}(\triangle ABC) = \frac{1}{2} AC \cdot BC = \frac{AC^2}{2} = \frac{25}{4}$ square units

33. (c) : Let $P(x) = 5x^4 + 4x^3 + 3x^2 + Mx + N$

Let $G(x) = x^2 + 1$

If the quotient is $Q(x)$, then $P(x) = Q(x)(x^2 + 1)$

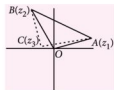
If $x = i$, then $P(i) = 0$

If $x = -i$, then $P(-i) = 0$

Hence, $5 - 4i - 3 + Mi + N = 0$

$\Rightarrow N + Mi = -2 + 4i$

$\therefore M - N = 6$



34. (b) : $(Z^3 + 3)^2 = 16i^2$

$\Rightarrow Z^3 + 3 = 4i$ or $-4i \Rightarrow Z^3 = -3 + 4i$ or $-3 - 4i$

$\Rightarrow |Z|^3 = |-3 + 4i| = 5 \quad \therefore |Z| = 5^{1/3}$

35. (c)

36. (c) : $z = \frac{at+b}{t-c} \Rightarrow t = \frac{b+cz}{z-a}$

Since, $|t| = 1 \Rightarrow t\bar{t} = 1 \Rightarrow \frac{(b+cz)(\bar{b}+\bar{c}\bar{z})}{(z-a)(\bar{z}-\bar{a})} = 1$

$\Rightarrow z\bar{z}(c\bar{c}-1) + (\bar{c}\bar{b}+a)z + (\bar{c}b+a)\bar{z} + b\bar{b}-a\bar{a} = 0$

37. (b) : Using quadratic formula, we get

$z = \frac{3i \pm \sqrt{-9+40k}}{20} \quad \therefore D = -9+40k.$

If $k = 1$, then $D = 31$, so (a) is false.

If k is a negative real number, then D is a negative real number, so (b) is true.

If $k = i$, then $D = -9 + 40i = 16 + 40i + 25i^2 = (4 + 5i)^2$,

and the roots are $\frac{1}{5} + \frac{2}{5}i$ and $-\frac{1}{5} - \frac{1}{10}i$, so (c) is false.

If $k = 0$ (which is a complex number), then the roots are

0 and $\frac{3}{10}i$, so (d) is false.

38. (d) : Refer the figure, z lies on the point of intersection of the rays from A and B . $\triangle ACB$ is a right angle and OBC is an equilateral triangle.

$\Rightarrow OC = a \Rightarrow$ argument of z is $\pi/3$

39. (b) : Let α is a non real complex root of unity that is also a root of the equation $z^2 + az + b = 0$, then $\bar{\alpha}$ will also be its root. ($|\alpha| = 1$)

Hence, $\alpha + \bar{\alpha} = -a$

$\therefore |a| = |\alpha + \bar{\alpha}| \leq |\alpha| + |\bar{\alpha}| = 2$

and $b = \alpha\bar{\alpha} = 1$

Hence we must check those equation for which $-2 \leq a \leq 2$ and $b = 1$

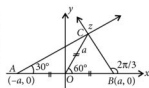
i.e. $z^2 + 2z + 1 = 0$

$\Rightarrow z^2 + z + 1 = 0 \Rightarrow z^2 + 1 = 0$

$\Rightarrow z^2 - 2z + 1 = 0 \Rightarrow z^2 - z + 1 = 0$

Hence roots are $\pm 1, \pm i$; $\frac{-1 \pm \sqrt{-3}}{2}, \frac{1 \pm \sqrt{-3}}{2}$ i.e. 8

40. (c)



Lower cutoff allows 32,000 to qualify for IITs

The IITs that were confident of filling seats across campuses with a small pool of 18,000-odd candidates were asked to lower their entry bar by the Union ministry of human resources and development (MHRD) on 14th June.

The extended list will be issued by IIT-Kanpur. The first merit list would have been inadequate to fill up all the seats.

With the cutoff falling to 90 from 126, there are now 31,988 qualified candidates, up from 18,138. The extended merit list includes 3,445 preparatory candidates. Earlier, without the preparatory course, there were 14,304 candidates. The number of qualified overseas citizen of India candidates also went up to 90, persons of Indian origin to 15 and foreign students to four. The IITs have a total student intake capacity of 11,279.

This has also made the IITs relax the score for other backward classes (OBCs), and while 5,555 candidates qualified without any handicap, another 1,345 were added to the OBC pool by relaxing the score of the last general category candidate by 10%. While 732 scheduled caste students and 243 scheduled tribe students qualified without any relaxation, more than thrice that number, in each category, made the cut after a 50% relaxation in scores. A total of 2,278 SC and 674 ST candidates qualified after a 50% relaxation in scores. That brought down the aggregate cut-off to 90 for general category students, 81 for OBC candidates, and 45 each for SC and ST students. The minimum score in each subject fell to 12 for open category, 11 for OBCs and six for SCs and STs. Minimum percent marks for preparatory course rank list for each subject fell to 2.5.

On 13th June, the joint admission board (JAB), comprising directors and top-ranking officials of various IITs, decided to not lower the cutoff for the JEE

EXTENDED MERIT LIST OF JEE (A)

Rise in number of students who have qualified for IITs	Qualified students category-wise			
	Category	June 14	June 10	Total (2018)
31,988 18,138 June 10, 2018 June 14, 2018 (16,062 boys, 2,076 girls) (27,809 boys, 4,179 girls)	General	8,954	8,794	17,748
	OBC	3,824	3,140	6,964
	SC	771	4,709	5,480
	ST	293	1,495	1,788
	Total	13,842	18,138	31,980
2017	Qualifying cutoff scores			
	Category	June 14	June 10	2017
50,455 (43,318 boys, 7,137 girls)	General	90 (12)*	126 (12)	75 (13)
	OBC-NCL	81 (11)	114 (11)	115 (10)
	SC	45 (6)	63 (6)	64 (6)
	ST	45 (6)	63 (6)	64 (6)
* Aggregate score (Subject-wise cutoff)				

(Advanced) merit list and also planned to conduct a simulation based on last year's data to see whether the present merit list is adequate to fill up the seats. Following the MHRD intervention, the board decided to release an extended merit list. "It was decided that an extended merit list of JEE (Advanced) 2018 qualified candidates be released by lowering the aggregate cutoff marks of the candidates, this will not affect the list of previously declared qualified candidates.



WB JEE

SOLVED PAPER 2018

CATEGORY-I (Q. 1 to Q. 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{4}$ marks. No answer will fetch 0 marks.

- The approximate value of $\sin 31^\circ$ is
(a) > 0.5 (b) > 0.6 (c) < 0.5 (d) < 0.4
- Let $f_1(x) = e^x$, $f_2(x) = e^{f_1(x)}$, ..., $f_{n+1}(x) = e^{f_n(x)}$ for all $n \geq 1$. Then for any fixed n , $\frac{d}{dx} f_n(x)$ is
(a) $f_n(x)$ (b) $f_n(x)f_{n-1}(x)$
(c) $f_n(x)f_{n-1}(x) \dots f_1(x)$ (d) $f_n(x) \dots f_1(x)e^x$
- The domain of definition of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$ is
(a) $(-\infty, -1) \cup (2, \infty)$
(b) $[-1, 1] \cup (2, \infty) \cup (-\infty, -2)$
(c) $(-\infty, 1) \cup (2, \infty)$
(d) $[-1, 1] \cup (2, \infty)$
- Let $f: [a, b] \rightarrow \mathbb{R}$ be differentiable on $[a, b]$ & $k \in \mathbb{R}$. Let $f(a) = 0 = f(b)$. Also let $J(x) = f'(x) + kf(x)$. Then
(a) $J(x) > 0$ for all $x \in [a, b]$
(b) $J(x) < 0$ for all $x \in [a, b]$
(c) $J(x) = 0$ has at least one root in (a, b)
(d) $J(x) = 0$ through (a, b)
- Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. Then $\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$
(a) does not exist (b) is $\frac{50}{3}$
(c) is $\frac{53}{3}$ (d) is $\frac{22}{3}$
- Let $f: [a, b] \rightarrow \mathbb{R}$ be such that f is differentiable in (a, b) , f is continuous at $x = a$ & $x = b$ and moreover $f(a) = 0 = f(b)$. Then
(a) there exists at least one point c in (a, b) such that $f'(c) = f(c)$
(b) $f'(x) = f(x)$ does not hold at any point in (a, b)
(c) at every point of (a, b) , $f'(x) > f(x)$
(d) at every point of (a, b) , $f'(x) < f(x)$
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$. Then
(a) $f''(0) = 0$
(b) $f''(c) = 0$ for some $c \in \mathbb{R}$
(c) if $c \neq 0$, then $f''(c) \neq 0$
(d) $f'(x) > 0$ for some $x \neq 0$
- If $\int e^{\sin x} \left[\frac{x \cos^3 x - \sin x}{\cos^2 x} \right] dx = e^{\sin x} f(x) + c$ where c is constant of integration, then $f(x) =$
(a) $\sec x - x$ (b) $x - \sec x$
(c) $\tan x - x$ (d) $x - \tan x$
- If $\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log f(x) + c$, where c is constant of integration, then $f(x) =$
(a) $\frac{2}{(b^2 - a^2) \sin 2x}$ (b) $\frac{2}{ab \sin 2x}$
(c) $\frac{2}{(b^2 - a^2) \cos 2x}$ (d) $\frac{2}{ab \cos 2x}$
- If $M = \int_0^{\pi/2} \frac{\cos x}{x+2} dx$, $N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2} dx$, then the value of $M - N$ is
(a) π (b) $\frac{\pi}{4}$ (c) $\frac{2}{\pi - 4}$ (d) $\frac{2}{\pi + 4}$

11. The value of the integral $I = \int_{1/2014}^{2014} \frac{\tan^{-1} x}{x} dx$ is
- (a) $\frac{\pi}{4} \log 2014$ (b) $\frac{\pi}{4} \log 2014$
 (c) $\pi \log 2014$ (d) $\frac{1}{2} \log 2014$
12. Let $I = \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx$. Then
- (a) $\frac{1}{2} \leq I \leq 1$ (b) $4 \leq I \leq 2\sqrt{30}$
 (c) $\frac{\sqrt{3}}{8} \leq I \leq \frac{\sqrt{2}}{6}$ (d) $1 \leq I \leq \frac{2\sqrt{3}}{2}$
13. The value of $I = \int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$, is
- (a) 1 (b) π (c) e (d) $\pi/2$
14. The value of $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \dots + \sec^2 \frac{n\pi}{4n} \right]$ is
- (a) $\log_e 2$ (b) $\frac{\pi}{2}$ (c) $\frac{4}{\pi}$ (d) e
15. The differential equation representing the family of curves $y^2 = 2d(x + \sqrt{d})$ where d is a parameter, is of
- (a) order 2 (b) degree 2
 (c) degree 3 (d) degree 4
16. Let $y(x)$ be a solution of $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ and $y(0) = -1$. Then $y(1)$ is equal to
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) -1
17. The law of motion of a body moving along a straight line is $x = \frac{1}{2}vt$, x being its distance from a fixed point on the line at time t and v is its velocity there. Then
- (a) acceleration f varies directly with x
 (b) acceleration f varies inversely with x
 (c) acceleration f is constant
 (d) acceleration f varies directly with t
18. Number of common tangents of $y = x^2$ and $y = -x^2 + 4x - 4$ is
- (a) 1 (b) 2 (c) 3 (d) 4
19. Given that n numbers of A.M.s are inserted between two sets of numbers $a, 2b$ and $2a, b$ where $a, b \in R$. Suppose further that the m^{th} means between these sets of numbers are same, then the ratio $a : b$ equals
- (a) $n - m + 1 : m$ (b) $n - m + 1 : n$
 (c) $n : n - m + 1$ (d) $m : n - m + 1$
20. If $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$ then the value of x is
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) 2
21. If $Z_r = \sin \frac{2\pi r}{11} - i \cos \frac{2\pi r}{11}$ then $\sum_{r=0}^{10} Z_r =$
- (a) -1 (b) 0 (c) i (d) -i
22. If z_1 and z_2 be two non zero complex numbers such that $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$, then the origin and the points represented by z_1 and z_2
- (a) lie on a straight line
 (b) form a right angled triangle
 (c) form an equilateral triangle
 (d) form an isosceles triangle
23. If $b_1 b_2 = 2(c_1 + c_2)$ and b_1, b_2, c_1, c_2 are all real numbers, then at least one of the equations $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ has
- (a) real roots
 (b) purely imaginary roots
 (c) roots of the form $a + ib$ ($a, b \in R, ab \neq 0$)
 (d) rational roots
24. The number of selection of n objects from $2n$ objects of which n are identical and the rest are different is
- (a) 2^n (b) 2^{n-1}
 (c) $2^n - 1$ (d) $2^{n-1} + 1$
25. If $(2 \leq r \leq n)$, then ${}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2}$ is equal to
- (a) $2 \cdot {}^nC_{r+2}$ (b) ${}^{n+1}C_{r+1}$
 (c) ${}^{n+2}C_{r+2}$ (d) ${}^{n+1}C_r$
26. The number $(101)^{100} - 1$ is divisible by
- (a) 10^4 (b) 10^6 (c) 10^8 (d) 10^{12}
27. If n is even positive integer, then the condition that the greatest term in the expansion of $(1 + x)^n$ may also have the greatest coefficient is
- (a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n}{n+1} < x < \frac{n+1}{n}$
 (c) $\frac{n+1}{n+2} < x < \frac{n+2}{n+1}$ (d) $\frac{n+2}{n+3} < x < \frac{n+3}{n+2}$

28. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$, Then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

- (a) A^2 (b) $A^2 - A + I_3$
(c) $A^2 - 3A + I_3$ (d) $3A^2 + 5A - 4I_3$

29. If $a_r = (\cos 2r\pi + i \sin 2r\pi)^{1/9}$, then the value of

$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(a) 1 (b) -1 (c) 0 (d) 2

30. If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$, then the value of

$\sum_{r=1}^n S_r$ is independent of

- (a) x only (b) y only
(c) n only (d) x, y, z and n

31. If the following three linear equations have a non-trivial solution, then

$$\begin{aligned} x + 4ay + az &= 0 \\ x + 3by + bz &= 0 \\ x + 2cy + cz &= 0 \end{aligned}$$

- (a) a, b, c are in A.P.
(b) a, b, c are in G.P.
(c) a, b, c are in H.P.
(d) $a + b + c = 0$

32. On R , a relation ρ is defined by $x\rho y$ if and only if $x - y$ is zero or irrational. Then

- (a) ρ is equivalence relation
(b) ρ is reflexive but neither symmetric nor transitive
(c) ρ is reflexive & symmetric but not transitive
(d) ρ is symmetric & transitive but not reflexive

33. On the set R of real numbers, the relation ρ is defined by $x\rho y, (x, y) \in R$

- (a) if $|x - y| < 2$ then ρ is reflexive but neither symmetric nor transitive.
(b) if $x - y < 2$ then ρ is reflexive and symmetric but not transitive.
(c) if $|x| \geq y$ then ρ is reflexive and transitive but not symmetric.
(d) if $x > |y|$ then ρ is transitive but neither reflexive nor symmetric.

34. If $f: R \rightarrow R$ be defined by $f(x) = e^x$ and $g: R \rightarrow R$ be defined by $g(x) = x^2$. The mapping $gof: R \rightarrow R$ be defined by $(gof)(x) = g(f(x)) \forall x \in R$, Then

- (a) gof is bijective but f is not injective
(b) gof is injective and g is injective
(c) gof is injective but g is not bijective
(d) gof is surjective and g is surjective

35. In order to get a head at least once with probability ≥ 0.9 , the minimum number of times a unbiased coin needs to be tossed is

- (a) 3 (b) 4 (c) 5 (d) 6

36. A student appears for tests I, II and III. The student is successful if he passes in tests I, II or I, III. The probabilities of the student passing in tests I, II and III are respectively p, q and $\frac{1}{2}$. If the probability of

the student to be successful is $\frac{1}{2}$. Then

- (a) $p(1 + q) = 1$ (b) $q(1 + p) = 1$
(c) $pq = 1$ (d) $\frac{1}{p} + \frac{1}{q} = 1$

37. If $\sin 60^\circ + \sin 40^\circ + \sin 20^\circ = 0$, then general value of θ is

- (a) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$
(c) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (d) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$
(n is integer)

38. If $0 \leq A \leq \frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2} \tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ is equal to

- (a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

39. Without changing the direction of the axes, the origin is transferred to the point $(2, 3)$. Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

- (a) $x^2 + y^2 + 4 = 0$ (b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 - 8x - 12y + 48 = 0$
(d) $x^2 + y^2 = 9$

40. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ is 2α . The equation of the locus of the point P is

- (a) $x^2 + y^2 + 4x + 6y + 9 = 0$
(b) $x^2 + y^2 - 4x + 6y + 9 = 0$
(c) $x^2 + y^2 - 4x - 6y + 9 = 0$
(d) $x^2 + y^2 + 4x - 6y + 9 = 0$

41. The point Q is the image of the point P(1, 5) about the line $y = x$ and R is the image of the point Q about the line $y = -x$. The circumcenter of the ΔPQR is

(a) (5, 1) (b) (-5, 1) (c) (1, -5) (d) (0, 0)

42. The angular points of a triangle are A(-1, -7), B(5, 1) and C(1, 4). The equation of bisector of the angle $\angle ABC$ is

(a) $x = 7y + 2$ (b) $7y = x + 2$
(c) $y = 7x + 2$ (d) $7x = y + 2$

43. If one of the diameter of the circle, given by the equation $x^2 + y^2 + 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is (2, -3), the radius of S is

(a) $\sqrt{41}$ unit (b) $3\sqrt{5}$ unit
(c) $5\sqrt{2}$ unit (d) $2\sqrt{5}$ unit

44. A chord AB is drawn from the point A(0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, and is extended to M such that $AM = 2AB$. The locus of M is

(a) $x^2 + y^2 - 8x - 6y + 9 = 0$
(b) $x^2 + y^2 + 8x + 6y + 9 = 0$
(c) $x^2 + y^2 + 8x - 6y + 9 = 0$
(d) $x^2 + y^2 - 8x + 6y + 9 = 0$

45. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 9y^2 = 9$, then the ratio $a^2 : b^2$ equals

(a) 8 : 1 (b) 1 : 8 (c) 9 : 1 (d) 1 : 9

46. Let A, B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, the slope of the line AB is

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

47. Let P(at^2 , 2at), Q, R(ar^2 , 2ar) be three points on a parabola, $y^2 = 4ax$. If PQ is the focal chord and PK, QR are parallel where the co-ordinates of K is (2a, 0), then the value of r is

(a) $\frac{t}{1-t^2}$ (b) $\frac{1-t^2}{t}$ (c) $\frac{t^2+1}{t}$ (d) $\frac{t^2-1}{t}$

48. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line through parallel to the y-axis meets the circle $x^2 + y^2 = 9$ at Q where P, Q are on the same side of

the x-axis. If R is a point on PQ such that $\frac{PR}{RQ} = \frac{1}{2}$, then locus of R is

(a) $\frac{x^2}{9} + \frac{9y^2}{49} = 1$ (b) $\frac{x^2}{49} + \frac{y^2}{9} = 1$
(c) $\frac{x^2}{9} + \frac{y^2}{49} = 1$ (d) $\frac{9x^2}{49} + \frac{y^2}{49} = 1$

49. A point P lies on a line through Q(1, -2, 3) and is parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$. If P lies on the plane $2x + 3y - 4z + 22 = 0$, then segment PQ equals to

(a) $\sqrt{42}$ units (b) $\sqrt{32}$ units
(c) 4 units (d) 5 units

50. The foot of the perpendicular drawn from the point (1, 8, 4) on the line joining the points (0, -11, 4) and (2, -3, 1) is

(a) (4, 5, 2) (b) (-4, 5, 2)
(c) (4, -5, 2) (d) (4, 5, -2)

CATEGORY-II (Q. 51 to Q. 65)

Only one answer is correct. Correct answer will fetch full marks. 2. Incorrect answer or any combination of more than one answer will fetch $-\frac{1}{2}$ marks. No answer will fetch 0 marks.

51. A ladder 20 ft long leans against a vertical wall. The top end slides downwards at the rate of 2 ft per second. The rate at which the lower end moves on a horizontal floor when it is 12 ft from the wall is

(a) $\frac{8}{3}$ (b) $\frac{6}{5}$ (c) $\frac{3}{2}$ (d) $\frac{17}{4}$

52. For $0 \leq p \leq 1$ and for any positive a, b; let $I(p) = (a + b)^p$, $J(p) = a^p + b^p$, then

(a) $I(p) > J(p)$
(b) $I(p) < J(p)$
(c) $I(p) > J(p)$ in $[0, \frac{p}{2}]$ & $I(p) > J(p)$ in $(\frac{p}{2}, \infty)$
(d) $I(p) < J(p)$ in $[\frac{p}{2}, \infty)$ & $I(p) < J(p)$ in $[0, \frac{p}{2}]$

53. Let $\vec{\alpha} = \hat{i} + \hat{j} + \hat{k}$, $\vec{\beta} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{\gamma} = -\hat{i} + \hat{j} - \hat{k}$ be three vectors. A vector $\vec{\delta}$, in the plane of $\vec{\alpha}$ and $\vec{\beta}$, whose projection on $\vec{\gamma}$ is $\frac{1}{\sqrt{3}}$, is given by

(a) $-\hat{i} - 3\hat{j} - 3\hat{k}$ (b) $\hat{i} - 3\hat{j} - 3\hat{k}$
(c) $-\hat{i} + 3\hat{j} + 3\hat{k}$ (d) $\hat{i} + 3\hat{j} - 3\hat{k}$

54. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three unit vectors such that $\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\gamma} = 0$ and the angle between $\vec{\beta}$ and $\vec{\gamma}$ is 30° . Then $\vec{\alpha}$ is

- (a) $2(\vec{\beta} \times \vec{\gamma})$ (b) $-2(\vec{\beta} \times \vec{\gamma})$
(c) $\pm 2(\vec{\beta} \times \vec{\gamma})$ (d) $(\vec{\beta} \times \vec{\gamma})$

55. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $\operatorname{Re}(z_1) > 0$ and $\operatorname{Im}(z_2) < 0$, then $\frac{z_1 + z_2}{z_1 - z_2}$ is

- (a) One (b) real and positive
(c) real and negative (d) purely imaginary

56. From a collection of 20 consecutive natural numbers, four are selected such that they are not consecutive. The number of such selections is

- (a) 284×17 (b) 285×17
(c) 284×16 (d) 285×16

57. The least positive integer n such that

$$\begin{pmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\ -\sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix}^n$$
 is an identity matrix of order 2 is

- (a) 4 (b) 8 (c) 12 (d) 16

58. Let ρ be a relation defined on N , the set of natural numbers, as $\rho = \{(x, y) \in N \times N : 2x + y = 41\}$. Then

- (a) ρ is an equivalence relation
(b) ρ is only reflexive relation
(c) ρ is only symmetric relation
(d) ρ is not transitive

59. If the polynomial

$$f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix},$$

then the constant term of $f(x)$ is

- (a) $2 - 3 \cdot 2^b + 2^{3b}$ (b) $2 + 3 \cdot 2^b + 2^{3b}$
(c) $2 + 3 \cdot 2^b - 2^{3b}$ (d) $2 - 3 \cdot 2^b - 2^{3b}$

60. A line cuts the x -axis at $A(5, 0)$ and the y -axis at $B(0, -3)$. A variable line PQ is drawn perpendicular to AB cutting the x -axis at P and the y -axis at Q . If AQ and BP meet at R , then the locus of R is

- (a) $x^2 + y^2 - 5x + 3y = 0$
(b) $x^2 + y^2 + 5x + 3y = 0$
(c) $x^2 + y^2 + 5x - 3y = 0$
(d) $x^2 + y^2 - 5x - 3y = 0$

61. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Let $B(1, 7)$ and $D(4, -2)$ be two points on the circle such that tangents at B and D meet at C . The area of the quadrilateral $ABCD$ is

- (a) 150 sq. units (b) 50 sq. units
(c) 75 sq. units (d) 70 sq. units

62. Let $f(x) = \begin{cases} -2 \sin x, & \text{if } x \leq -\frac{\pi}{2} \\ A \sin x + B, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x, & \text{if } x \geq \frac{\pi}{2} \end{cases}$. Then

- (a) f is discontinuous for all A and B
(b) f is continuous for all $A = -1$ and $B = 1$
(c) f is continuous for all $A = 1$ and $B = -1$
(d) f is continuous for all real values of A, B .

63. The normals to the curve $y = x^2 - x + 1$, drawn at the points with the abscissa $x_1 = 0$, $x_2 = -1$ and

- $x_3 = \frac{5}{2}$
(a) are parallel to each other
(b) are pair wise perpendicular
(c) are concurrent
(d) are not concurrent

64. The equation $x \log x = 3 - x$

- (a) has no root in $(1, 3)$
(b) has exactly one root in $(1, 3)$
(c) $x \log x - (3 - x) > 0$ in $[1, 3]$
(d) $x \log x - (3 - x) < 0$ in $[1, 3]$

65. Consider the parabola $y^2 = 4x$. Let P and Q be points on the parabola where $P(4, -4)$ & $Q(9, 6)$. Let R be a point on the arc of the parabola between P & Q . Then the area of ΔPQR is largest when

- (a) $\angle PQR = 90^\circ$ (b) $R(4, 4)$
(c) $R\left(\frac{1}{4}, 1\right)$ (d) $R\left(1, \frac{1}{4}\right)$

CATEGORY-III (Q. 66 to Q. 75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = $2 \times \text{number of correct answers marked} \div \text{actual number of correct answers}$.

66. Let $I = \int_0^1 \frac{x^3 \cos 3x}{2 + x^2} dx$. Then

- (a) $-\frac{1}{2} < I < \frac{1}{2}$ (b) $-\frac{1}{3} < I < \frac{1}{3}$
(c) $-1 < I < 1$ (d) $-\frac{2}{3} < I < \frac{3}{2}$

67. A particle is in motion along a curve $12y = x^3$. The rate of change of its ordinate exceeds that of abscissa in

- (a) $-2 < x < 2$ (b) $x = \pm 2$
(c) $x < -2$ (d) $x > 2$

68. The area of the region lying above x -axis, and included between the circle $x^2 + y^2 = 2ax$ & the parabola $y^2 = ax$, $a > 0$ is

- (a) $8\pi a^2$ (b) $a^2 \left(\frac{\pi}{4} - \frac{2}{3} \right)$
(c) $\frac{16\pi a^2}{9}$ (d) $\pi \left(\frac{27}{8} + 3a^2 \right)$

69. If the equation $x^2 - cx + d = 0$ has roots equal to the fourth powers of the roots of $x^2 + ax + b = 0$, where $a^2 > 4b$, then the roots of $x^2 - 4bx + 2b^2 - c = 0$ will be

- (a) both real
(b) both negative
(c) both positive
(d) one positive and one negative

70. On the occasion of Dipawali festival, each student of a class sends greeting cards to others. If there are 20 students in the class, the number of cards sent by students is

- (a) ${}^{20}C_2$ (b) ${}^{20}P_2$
(c) $2 \times {}^{20}C_2$ (d) $2 \times {}^{20}P_2$

71. In a third order matrix A , a_{ij} denotes the element in the i -th row and j -th column.

- If $a_{ij} = 0$ for $i = j$
= 1 for $i > j$
= -1 for $i < j$

Then the matrix is

- (a) skew symmetric (b) symmetric
(c) not invertible (d) non-singular

72. The area of the triangle formed by the intersection of a line parallel to x -axis and passing through $P(h, k)$, with the lines $y = x$ and $x + y = 2$ is h^2 . The locus of the point P is

- (a) $x = y - 1$ (b) $x = -(y - 1)$
(c) $x = 1 + y$ (d) $x = -(1 + y)$

73. A hyperbola, having the transverse axis of length $2 \sinh$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Its equation is

- (a) $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$
(b) $x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$
(c) $(x^2 + y^2) \sin^2 \theta = 1 + y^2$
(d) $x^2 \operatorname{cosec}^2 \theta = x^2 + y^2 + \sin^2 \theta$

74. Let $f(x) = \cos\left(\frac{\pi}{x}\right)$, $x \neq 0$, then assuming k as an integer,

- (a) $f(x)$ increases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$
(b) $f(x)$ decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k}\right)$
(c) $f(x)$ decreases in the interval $\left(\frac{1}{2k+1}, \frac{1}{2k+1}\right)$
(d) $f(x)$ increases in the interval $\left(\frac{1}{2k+2}, \frac{1}{2k+1}\right)$

75. Consider the function $y = \log_a(x + \sqrt{x^2 + 1})$, $a > 0$, $a \neq 1$. The inverse of the function

- (a) does not exist
(b) is $x = \log_{1/a}(y + \sqrt{y^2 + 1})$
(c) is $x = \sinh(y \ln a)$
(d) is $x = \cosh\left(-y \ln \frac{1}{a}\right)$

SOLUTIONS

1. (a) : $\because \frac{1}{2} = \sin 30^\circ < \sin 31^\circ$ (slightly)

$$\therefore \sin 31^\circ > 0.5$$

$$\begin{aligned} 2. (c) : \frac{d}{dx}\{f_n(x)\} &= \frac{d}{dx}\{e^{f_{(n-1)}(x)}\} \\ &= e^{f_{n-1}(x)} \cdot \frac{d}{dx}\{f_{n-1}(x)\} = f_n(x) \cdot \frac{d}{dx}\{e^{f_{n-2}(x)}\} \\ &= f_n(x) \cdot e^{f_{n-2}(x)} \cdot \frac{d}{dx}\{f_{n-2}(x)\} \end{aligned}$$

$\therefore$ Proceeding in the same way,

Required value $= f_n(x)f_{n-1}(x) \dots f_1(x)$

3. (b) : For $f(x)$ to be defined,

$$\frac{1-|x|}{2-|x|} \geq 0 \Rightarrow \frac{|x|-1}{|x|-2} \geq 0$$

$$\Rightarrow |x| \leq 1 \text{ or } |x| > 2 \Rightarrow -1 \leq x \leq 1 \text{ or } \{x < -2 \text{ or } x > 2\}$$

$$\Rightarrow D_f = (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$$

$$\begin{aligned} 4. (c) : \because \frac{d}{dx}\{f(x) \cdot e^{kx}\} &= f'(x)e^{kx} + f(x) \cdot ke^{kx} \\ &= e^{kx}\{f'(x) + kf(x)\} \end{aligned}$$

$$\text{Let } g(x) = f(x)e^{kx}$$

$$\therefore g(a) = f(a) \cdot e^{ka} = 0 \quad [\because f(a) = 0]$$

$$g(b) = f(b)e^{kb} = 0 \quad [\because f(b) = 0]$$

Being product of 2 continuous functions, $g(x)$ is cont. in $[a, b]$

Also $g'(x) = e^{kx}(f''(x) + kf(x)) = e^{kx}J(x)$, which exists
 $\therefore$ All conditions of Rolle's Theorem are satisfied.

$\therefore g'(c) = 0$ for at least one $c \in (a, b)$

$\Rightarrow e^{kc} \cdot J(c) = 0$ for at least one $c \in (a, b)$

$\Rightarrow J(c) = 0$ has at least one root in (a, b)

5. (c) : $f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$f'(1) = -53$... (1)

$$\therefore \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h} = \lim_{h \rightarrow 0} \left[\frac{f(1-h) - f(1)}{(-h)} \cdot \frac{-1}{(h^2 + 3)} \right]$$

$$= f'(1) \left(-\frac{1}{3} \right) = (-53) \left(-\frac{1}{3} \right) = \frac{53}{3}$$

6. (a) : Let us assume, $g(x) = e^{-x}f(x) \therefore f'(c) - f(c)$ or $f'(x) - f(x)$ expression gives us following :

$$f'(x) - f(x) = 0$$

$$\Rightarrow e^{-x}f'(x) + e^{-x}(-1)f(x) = 0 \Rightarrow \frac{d}{dx} [e^{-x}f(x)] = 0$$

From given condition and nature of e^{-x} , we can say that

(i) $g(x)$ is continuous in $[a, b]$

(ii) $g(x)$ is differentiable in (a, b) and

and (iii) $g(a) = e^{-a}f(a) = 0$ and $g(b) = e^{-b}f(b) = 0$

$$[\therefore f(a) = 0 = f(b)]$$

From Rolle's theorem, $g'(c) = 0$ for at least one $c \in (a, b)$

$$\Rightarrow e^{-c}f'(c) - e^{-c}f(c) = 0$$

$$\Rightarrow f'(c) - f(c) = 0 \quad [\therefore e^{-c} \neq 0]$$

$$\Rightarrow f'(c) = f(c) \text{ for at least one } c \in (a, b)$$

7. (b) : $\therefore f$ is a twice continuously differentiable function

$\therefore f(x)$ and $f'(x)$ both are continuous and differentiable functions.

$$\therefore f(0) = f(1) = 0$$

Using Rolle's Theorem, $f'(c_1) = 0$ for some $c_1 \in (0, 1)$

$$\text{Further } f'(0) = 0 = f'(c_1)$$

Using Rolle's Theorem, $f''(c) = 0$ for some $c \in (0, c_1)$

$$\Rightarrow f''(c) = 0 \text{ for some } c \in \mathbb{R}$$

$$8. (b) : \int e^{\sin x} \left(\frac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx$$

$$= \int e^{\sin x} (x \cos x) dx - \int e^{\sin x} (\sec x \tan x) dx$$

$$= \int x \cdot (e^{\sin x} \cdot \cos x) dx - [e^{\sin x} \cdot \sec x$$

$$- \int (e^{\sin x} \cos x) \sec x dx]$$

(Using by parts)

$$= x(e^{\sin x}) - \int 1 \cdot e^{\sin x} dx - e^{\sin x} \cdot \sec x + \int e^{\sin x} dx$$

$$= e^{\sin x}(x - \sec x) + c \Rightarrow f(x) = x - \sec x$$

$$9. (c) : \int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log[f(x)] + c$$

On differentiating, both sides, we get

$$f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow \int \frac{f'(x) dx}{[f(x)]^2} = \int (b^2 - a^2) \sin 2x dx$$

$$\Rightarrow \frac{-1}{f(x)} = (b^2 - a^2) \left(-\frac{\cos 2x}{2} \right) \Rightarrow f(x) = \frac{2}{(b^2 - a^2) \cos 2x}$$

$$10. (d) : N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x+1)^2} dx$$

$$\text{Put } 2x = z \Rightarrow dx = \frac{1}{2} dz$$

x	0	$\pi/4$
z	0	$\pi/2$

$$\therefore N = \int_0^{\pi/2} \frac{\sin z}{4 \left(\frac{z}{2} + 1 \right)^2} dz = \int_0^{\pi/2} \frac{\sin z}{(z+2)^2} dz$$

$$= \left[\sin z \left(-\frac{1}{z+2} \right) \right]_0^{\pi/2} - \int_0^{\pi/2} \cos z \left(-\frac{1}{z+2} \right) dz$$

$$= \frac{-2}{\pi+4} + \int_0^{\pi/2} \frac{\cos x}{x+2} dx = \frac{-2}{\pi+4} + M$$

$$\therefore M - N = \frac{2}{\pi+4}$$

- | | | | | |
|------------------|------------|------------|------------|---------|
| 11. (b) | 12. (c) | 13. (b) | 14. (c) | 15. (c) |
| 16. (c) | 17. (c) | 18. (b) | 19. (d) | 20. (c) |
| 21. (b) | 22. (c) | 23. (a) | 24. (a) | 25. (c) |
| 26. (a) | 27. (a) | 28. (a) | 29. (c) | 30. (d) |
| 31. (c) | 32. (c) | 33. (d) | 34. (c) | 35. (b) |
| 36. (a) | 37. (a) | 38. (b) | 39. (b) | 40. (d) |
| 41. (d) | 42. (b) | 43. (a) | 44. (c) | 45. (a) |
| 46. (c, d) | 47. (d) | 48. (a) | 49. (a) | 50. (d) |
| 51. (a) | 52. (b) | 53. (c) | 54. (c) | 55. (d) |
| 56. (a) | 57. (b) | 58. (d) | 59. (a) | 60. (a) |
| 61. (c) | 62. (b) | 63. (c) | 64. (b) | 65. (c) |
| 66. (a, b, c, d) | | 67. (c, d) | 68. (b) | |
| 69. (a, d) | 70. (b, c) | 71. (a, c) | 72. (a, b) | 73. (b) |
| 74. (a, c) | | 75. (c) | | |

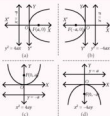
For complete solutions refer to

MTG WB JEE Chapterwise Explorer (Mathematics)



Definition

Parabola is locus of point which moves in a plane such that its distance from a fixed line is equal to its distance from a fixed point. The fixed point is the focus. The fixed line is the directrix.



Equation of Tangent

Curve	Point	Equation of Tangent
$y^2 = 4ax$	(x_1, y_1)	$yy_1 = 2a(x + x_1)$ (Point form)
	$(at^2, 2at)$	$ty = x + at^2$ (Parametric form)
	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ (slope is given)	$y = mx + \frac{a}{m}$ (Slope form)

Equation of Normal

Curve	Point	Condition
$y^2 = 4ax$	(x_1, y_1)	$y - y_1 = -\frac{y_1}{2a}(x - x_1)$ (One point form)
	$(at^2, 2at)$	$y + tx = at^3 + 2at$ (Parametric form)
	$(am^2, -2am)$ (slope form)	$y = mx - 2am - am^3$ (Slope form)

Standard Form

Standard form of parabola is $y^2 = 4ax, a > 0$

- Focus (F) : $(a, 0)$
- Vertex : $(0, 0)$
- Directrix (L) : $x = -a$
- Axis : $y = 0$
- Tangent at vertex : $x = 0$
- Latus Rectum, L_1L_2 : $x = a$
- Length of latus rectum = $4a$



Condition of Tangency of a Line to the Parabola

Parabola	Line	Condition
$y^2 = 4ax$	$y = mx + c$	$c = \frac{a}{m}$
	$x \cos \alpha + y \sin \alpha = p$	$p = -a \tan \alpha \sin \alpha$
	$lx + my + n = 0$	$ln = am^2$

Some Important Results

- Locus of point of intersection of perpendicular tangents to the parabola is its directrix, $x = -a$.
- Equation of chord of parabola bisected at point (x_1, y_1) is given by $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$.
- Equation of chord of contact (QR) of tangents to a parabola :
 - Equation of QR is given by $yy_1 = 2a(x + x_1)$
 - If $Q(at_1^2, 2at_1)$, $R(at_2^2, 2at_2)$ then equation of QR is given by $y(t_1 + t_2) = 2x + 2at_1t_2$
- The length of chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle θ with the axis is $4a \cot \theta$ cosec θ .



Definition

A relation $f: A \rightarrow B$ a function where every element of set A has only one image in set B.

Invertible Functions

Let $f: A \rightarrow B$ be a bijection. Then inverse function is defined as $f^{-1}: B \rightarrow A$ i.e. $f^{-1}(b) = a \Leftrightarrow f(a) = b$.

Arrow diagram :



Composition of Functions

For two functions $f: X \rightarrow Y$ and $g: Y \rightarrow Z$, composition of functions is denoted by

$g \circ f$ defined as

$g \circ f(x) = g(f(x)) \forall x \in X$.

Properties

- Composition of functions is not commutative, i.e., $f \circ g \neq g \circ f$ (in general).
- Composition of functions is associative i.e., $f \circ (g \circ h) = (f \circ g) \circ h$
- Composition of two bijections is also a bijection.

Number of Functions

Let $f: A \rightarrow B$ such that $n(A) = m$, $n(B) = n$.

- The number of functions is n^m .
- The number of one-one functions are ${}^nP_m = \frac{n!}{(n-m)!}$, if $m \leq n$.
- The number of onto functions are $n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \binom{n}{3}(n-3)^m + \dots$, if $m \geq n$.
- The number of bijections is $n!$, if $m = n$.

Types of Functions

Type	Definition	Representation
One-one (Injective) Function	A function $f: X \rightarrow Y$ is one-one, if different elements of X have different images in Y under f. (i) $n(X) \leq n(Y)$ (ii) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	
Onto (Surjective) Function	A function $f: X \rightarrow Y$ is onto, if every element of Y is the image of atleast one element of X under f. (i) $n(X) \geq n(Y)$ (ii) Range = Co-domain	
Many-one Function	A function $f: X \rightarrow Y$ is many-one, if two or more than two elements of X have the same image in Y.	
Into Function	A function $f: X \rightarrow Y$ is into, if there exists a single element in Y having no pre-image in X.	
Bijjective Function	A function $f: X \rightarrow Y$ is bijective, if it is both one-one and onto i.e., $n(X) = n(Y)$ and Range = Co-domain	

Analysis of Functions by Graphical Method

- If any line parallel to x axis intersects graph of $y = f(x)$ not more than once, then it is one-one.
- If $f'(x)$ is always strictly increasing or decreasing then it is one-one.
- If atleast one line parallel to the x axis intersects more than once or $f'(x) = 0$ over a continuous interval or changes its sign, then it is onto.



MOCK TEST PAPER 2019 JEE MAIN

Time : 1 hr 15 min.

Series - 2

The entire syllabus of Mathematics of JEE MAIN is being divided into eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus In Detail
UNIT NO.2	Complex Numbers	Algebra of complex numbers, addition, multiplication, conjugation, polar representation, properties of modulus and principal argument, triangle inequality, square root of a complex number cube roots of unity, geometric interpretations.
	Quadratic Equations	Quadratic equations in real and complex system and their solutions. Quadratic equations with real coefficients, relation between roots and coefficients, formation of quadratic equation s with given roots, symmetric functions of roots.
	Trigonometry	Compound angles, Transformation of sum & product, Multiple & sub-multiple angles.
	Co-ordinate Geometry-2D	Straight lines: Equation of a straight line in various forms, angle between two lines, distance of a point from a line, lines through the point of intersection of two given lines, equation of the bisector of the angle between two lines, concurrency of lines, centroid, orthocenter, incentre and circumcentre of a triangle.

1. The locus of $z = a + ib$ which satisfy the inequality $\log_1 \frac{|z-1|}{2} > \log_1 \frac{|z-i|}{2}$ is given by

- (a) $a + b < 0$ (b) $a - b > 0$
(c) $a - b < 0$ (d) $a + b > 0$

2. If $\alpha = \cos \frac{8\pi}{11} + i \sin \frac{8\pi}{11}$, then

- $\text{Re}(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5) =$
(a) 0 (b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) none of these

3. If a, b, c are positive integer and

- $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$, then $f(\omega) =$
(a) 0 (b) 1 (c) -1 (d) i

4. Let z and ω are two non-zero complex numbers such that $|z| = |\omega|$ and $\arg(z) + \arg(\omega) = \pi$, then z equals

- (a) $\bar{\omega}$ (b) ω (c) $-\bar{\omega}$ (d) $-\omega$

5. If $S = i + 2i^2 + 3i^3 + \dots$ up to 200 terms, then $S =$

- (a) $200i$ (b) $100(1+i)$
(c) $100(1-i)$ (d) $200(1-i)$

6. The range of $|z|$, when z satisfies the condition $\left| z - \frac{4}{z} \right| = 2$, is

- (a) $[\sqrt{5}-1, \sqrt{5}]$ (b) $1-\sqrt{5} \leq |z| \leq 1+\sqrt{5}$
(c) $-\sqrt{5} \leq |z| \leq \sqrt{5}$ (d) none of these

7. Let $z = (1-t) + i\sqrt{t^2+t+2}$, where t is real parameter then locus of z in the Gaussian plane is

- (a) a straight line (b) an ellipse
(c) a hyperbola (d) a circle

8. If the area of the triangle on the complex plane formed by the points $z, iz, z+iz$ is 200 square units then $|z| =$

- (a) 15 (b) 10 (c) 200 (d) 20

9. If $p, q, r \in \mathbb{R}$, then the roots of the equation $p(x^2 - z^2) - (q-r)xz = 0$ is

- (a) imaginary
(b) real roots but not equal
(c) Sum of the roots is zero
(d) none of these

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10. If the equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ have one root in common, then the value of $a^3 + b^3 + c^3 =$

- (a) $3a(a + b + c)$
(b) $a^2 + b^2 + c^2 - ab - bc - ca$
(c) $3(ab + bc + ca)$ (d) $3abc$

11. If α, β are the roots of the equation $6x^2 - x - 2 = 0$, then the equation whose roots are $\alpha^2 + 2, \beta^2 + 2$ is

- (a) $36x^2 - 169x + 198 = 0$
(b) $36x^2 + 169x + 198 = 0$
(c) $36x^2 - 196x - 198 = 0$
(d) $x^2 - 169x + 198 = 0$

12. If the roots of the equation $x^2 - \omega x + v = 0$ differ by 2, then

- (a) $v^2 = 4(1 + \omega)$ (b) $v^2 = 1(1 - \omega)$
(c) $\omega^2 = 4(1 + v)$ (d) $\omega^2 = 4(1 - v)$

13. If α, β are the roots of the equation $x^2 + px + q = 0$ and α^4, β^4 are the roots of the equation $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always

- (a) both positive roots (b) both negative roots
(c) real and distinct roots (d) non-real roots

14. The set of value of x , for which the inequality $[x]^2 - 5[x] + 6 \leq 0$ [where $[\cdot]$ denotes the greatest integral function] hold good if

- (a) $2 \leq [x] \leq 5$ (b) $2 \leq [x] < 4$
(c) $2 \leq [x] \leq 3$ (d) both (b) and (c)

15. If a, b, c be the $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms respectively of an A.P. and G.P. both, then the product of the roots of the equation $(a^b b^c c^a)x^2 - (abc)x + (a^c b^a c^b) = 0$ equals

- (a) -1 (b) 2 (c) abc (d) 1

16. Let $y = \frac{mx^2 + 3x - 4}{m + 3x - 4x^2}$, the values of m for which

$y \in R, \forall x \in R$ is

- (a) $-7 \leq m \leq -1$ (b) $m \geq -4$
(c) $1 \leq m \leq 7$ (d) none of these

17. For $0 < \theta < \frac{\pi}{2}$, $\tan \theta + \tan 2\theta = -\tan 3\theta$, if

- (a) $\tan \theta = 0$ (b) $\tan 2\theta = 0$
(c) $\frac{1}{\tan 3\theta} = 0$ (d) $\tan \theta \tan 2\theta = -2$

18. In a triangle one angle is $\frac{3\pi}{4}$ and the other angles are two values of θ satisfying $a \tan \theta + b \sec \theta = c$, where $|b| \leq \sqrt{a^2 + c^2}$. Then $a^2 - c^2 =$

- (a) ac (b) $2ac$ (c) $\frac{a}{c}$ (d) $\frac{2a}{c}$

19. The value of $\tan^6 40^\circ - 33 \tan^4 40^\circ + 27 \tan^2 40^\circ$ is

- (a) 1 (b) 2
(c) 3 (d) none of these

20. Two parallel chords are drawn on the same side of the centre of a circle of radius 12 units. It is found that they subtend 72° and 144° at the centre of the circle. The perpendicular distance (in units) between the chords is

- (a) 6 (b) 4 (c) 3 (d) 2

21. If $f(x) = \sin^2 x + \sin^2 \left(x + \frac{\pi}{3}\right) + \cos x \cos \left(x + \frac{\pi}{3}\right)$,

$g\left(\frac{5}{4}\right) = 1$, then $gf\left(\frac{\pi}{8}\right) =$

- (a) $\frac{5}{4}$ (b) 1 (c) 2 (d) π

22. $\cos^2 A + \cos^2(B - A) - 2 \cos A \cos B \cos(A - B) =$

- (a) $\cos^2 A$ (b) $\sin^2 A$
(c) $\sin^2 B$ (d) $\cos^2 B$

23. A variable straight line passes through the intersection of the lines $x + 2y = 1$ and $2x - y = 1$ meet the co-ordinate axes A and B , then the locus of middle point of AB is

- (a) $x - 3y = 10xy$ (b) $x + 3y = 10xy$
(c) $3x + y = 10xy$ (d) none of these

24. The lines $2x + y - 1 = 0$, $ax + 3y - 3 = 0$ and $3x + 2y - 2 = 0$ are concurrent, then a satisfies the condition

- (a) a is any real number (b) $a \in [-2, 3]$
(c) $a \geq 0$ (d) $a \leq 0$

25. The image of the point $(2, 3)$ in the line $x + 3y + 4 = 0$ is

- (a) $(1, 6)$ (b) $(-1, -6)$
(c) $(-1, 6)$ (d) $(1, -6)$

26. The equation of the bisectors of the angles between the lines $3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$, if positive sign is taken for the bisector is

- (a) $99x + 77y - 131 = 0$ (b) $21x - 27y - 131 = 0$
(c) $21x + 27y - 131 = 0$ (d) $9x - 77y - 131 = 0$

27. If R_1 and R_2 be the perpendicular drawn

from the origin to the line $\frac{x}{\cos \theta} + \frac{y}{\sin \theta} = a$

and $\frac{x}{\sec \theta} - \frac{y}{\csc \theta} = \frac{a}{\sec 2\theta}$ respectively then the value

of the expression $4R_1^2 + R_2^2$ equals

- (a) $4a^2$ (b) $3a^2$ (c) $2a^2$ (d) a^2

28. The equation of the straight line which passes through the point $(-5, 4)$ and is such that the portion of it between the axes is divided by the given in the ratio 1:2 is

- (a) $\frac{x}{12} + \frac{y}{-15} = 1$ (b) $\frac{x}{-15} + \frac{y}{12} = 1$
 (c) $\frac{x}{12} + \frac{y}{15} = 1$ (d) none of these

29. The lines joining the origin to the point of intersection of $3x - 4y = 56$ and the curve $x^2 + y^2 - 24x + 10y = 0$ is

- (a) coincident lines (b) lines are at 90°
 (c) equal in length
 (d) makes equal angles with y -axis

30. The nearest point on the line $3x + 4y = 25$ from the origin is

- (a) $(-5, 5)$ (b) $(3, 4)$
 (c) $(3, 5)$ (d) $(3, -4)$

SOLUTIONS

1. (b): Here the given inequality is

$$\log_1 |z-1| > \log_1 |z-i|$$

Here the base is $\frac{1}{2} \in (0, 1)$

$$\therefore |z-1| < |z-i| \Rightarrow |z-1|^2 < |z-i|^2$$

$$\Rightarrow (z-1)(\bar{z}-1) < (z-i)(\bar{z}+i) \quad [\because |z|^2 = z\bar{z}]$$

$$\Rightarrow z\bar{z} - \bar{z} - z + 1 < z\bar{z} - i\bar{z} + zi - i^2$$

$$\Rightarrow i\bar{z} - \bar{z} - z - zi < 0 \Rightarrow z + \bar{z} + zi - \bar{z}i > 0$$

$$\Rightarrow (z + \bar{z}) + (z - \bar{z})i > 0 \Rightarrow \left(\frac{z + \bar{z}}{2}\right) + i\left(\frac{z - \bar{z}}{2}\right) > 0$$

$$\Rightarrow \text{Re}(z) - \text{Im}(z)i > 0 \Rightarrow a - b > 0 \quad [\text{since } z = a + ib]$$

2. (b): We know that $\text{Re}(z) = \frac{z + \bar{z}}{2}$

$$\therefore \text{Re}(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5) = \frac{(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5) + (\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)}{2}$$

$$= \frac{1}{2} \left(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + \frac{1}{\alpha^3} + \frac{1}{\alpha^4} + \frac{1}{\alpha^5} \right)$$

$$= \frac{1}{2\alpha^5} (1 + \alpha + \alpha^2 + \alpha^3 + \dots + \alpha^{10} - \alpha^5)$$

$$= \frac{1}{2\alpha^5} \left[\frac{1 - \alpha^{11}}{1 - \alpha} - \alpha^5 \right] = \frac{1}{2\alpha^5} [0 - \alpha^5] = -\frac{1}{2}$$

$$\therefore \alpha = \cos \frac{8\pi}{11} + i \sin \frac{8\pi}{11} \Rightarrow \alpha^{11} = \cos 8\pi + i \sin 8\pi = 1$$

3. (a): Here $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$

$$\therefore f(\omega) = (\omega^6)^a + \omega(\omega^6)^b + \omega^2(\omega^6)^c$$

$$\Rightarrow f(\omega) = 1 + \omega + \omega^2 = 0$$

4. (c): Given that $|z| = |\omega| = r$, let $\arg(\omega) = \theta$

$$\therefore \omega = r(\cos\theta + i\sin\theta)$$

$$\text{and } \arg(z) + \arg(\omega) = \pi \Rightarrow \arg(z) = \pi - \theta$$

$$\therefore z = r(\cos(\pi - \theta) + i\sin(\pi - \theta)) = r(-\cos\theta + i\sin\theta) = -r(\cos\theta - i\sin\theta) = -\bar{\omega}$$

5. (c): Here $S = i + 2i^2 + 3i^3 + \dots$

$$\Rightarrow iS = i^2 + 2i^3 + \dots + 198i^{199} + 199i^{200} + 200i^{201} \dots (1)$$

$$\text{Subtract eqn. (1) from eqn. (2), we get}$$

$$S(1 - i) = i + i^2 + i^3 + \dots + i^{200} - 200i^{201} = 0 - 200i^{201}$$

$$\text{Therefore, } S = \frac{-200i}{(1-i)} = \frac{-200i(1+i)}{2} = 100(1-i).$$

6. (b): We may write, $|z| = \left| z - \frac{4}{z} + \frac{4}{z} \right|$

$$\Rightarrow |z| \leq \left| z - \frac{4}{z} \right| + \left| \frac{4}{z} \right| \Rightarrow |z| \leq 2 + \frac{4}{|z|} \quad \left[\because \left| z - \frac{4}{z} \right| = 2 \right]$$

$$\Rightarrow |z|^2 - 2|z| - 4 \leq 0 \Rightarrow |z|^2 - 2|z| + 1 \leq 5$$

$$\Rightarrow (|z| - 1)^2 - (\sqrt{5})^2 \leq 0$$

$$\Rightarrow (|z| - 1 + \sqrt{5})(|z| - 1 - \sqrt{5}) \leq 0$$

$$\Rightarrow 1 - \sqrt{5} \leq |z| \leq 1 + \sqrt{5}$$

7. (c): Given that $z = (1-t) + i\sqrt{t^2 + t + 2}$

$$\text{Let } z = x + iy = (1-t) + i\sqrt{t^2 + t + 2}$$

$$\Rightarrow x = 1 - t \quad \dots (1)$$

$$\text{and } y^2 = t^2 + t + 2 = \left(t + \frac{1}{2}\right)^2 + \frac{7}{4} \quad \dots (2)$$

$$\text{Now from (1), we get } t + \frac{1}{2} = \frac{3-x}{2}$$

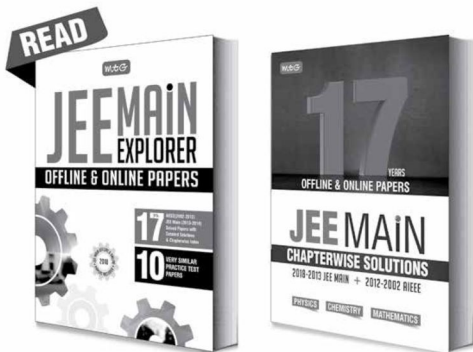
Now, eqn. (2) gives

$$y^2 = \left(\frac{3-x}{2}\right)^2 + \frac{7}{4} \Rightarrow y^2 - \frac{7}{4} = \left(\frac{3-x}{2}\right)^2$$

$$\Rightarrow -\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{7}{4}$$

which represents a hyperbola.

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8. (d): The area of the triangle formed by z , iz , $z + iz$ is always given by $\frac{1}{2}|z|^2$.

$$\therefore \frac{1}{2}|z|^2 = 200$$

$$\Rightarrow |z|^2 = 400 \Rightarrow |z| = 20$$

9. (b): The given equation is

$$p(x^2 - z^2) - (q - r)xz = 0$$

$$\Rightarrow px^2 - (q - r)xz - pz^2 = 0$$

$$\Rightarrow p\left(\frac{x}{z}\right)^2 - (q - r)\frac{x}{z} - p = 0$$

Now, $D = (q - r)^2 + 4p^2 > 0$. Thus the roots are real and distinct.

10. (d): Let the common root be α

$$\therefore a\alpha^2 + b\alpha + c = 0 \text{ and } c\alpha^2 + b\alpha + a = 0$$

$$\Rightarrow \frac{\alpha^2}{ba - bc} = \frac{\alpha}{c^2 - a^2} = \frac{1}{ab - cb}$$

$$\therefore (c^2 - a^2)^2 = (ba - bc)(ba - bc)$$

(the condition for common root)

$$\Rightarrow (c + a)^2(c - a)^2 = (ba - bc)^2$$

$$\Rightarrow (c + a)^2(c - a)^2 = b^2(c - a)^2$$

$$\Rightarrow (c - a)^2\{(c + a)^2 - b^2\} = 0$$

$$\Rightarrow c = a \text{ or } (c + a)^2 - b^2 = 0$$

$$\Rightarrow c = a \text{ or } (a + b + c)(a - b + c) = 0$$

$$\Rightarrow c = a \text{ or } (a + b + c) = 0 \text{ or } (a - b + c) = 0$$

Now from $a + b + c = 0$ we get $a + b = -c$

$$\Rightarrow (a + b)^3 = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(a + b) = -c^3$$

$$\Rightarrow a^3 + b^3 + 3ab(-c) = -c^3$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

11. (a): Since, α, β are the roots of the equation $6x^2 - x - 2 = 0$

$$\therefore \alpha + \beta = \frac{1}{6} \text{ and } \alpha\beta = -\frac{1}{3}$$

$$\text{Now, } \alpha^2 + 2 + \beta^2 + 2 = \alpha^2 + \beta^2 + 4$$

$$= (\alpha + \beta)^2 - 2\alpha\beta + 4$$

$$= \frac{1}{36} - 2 \times \left(-\frac{1}{3}\right) + 4 = \frac{169}{36}$$

$$\text{And } (\alpha^2 + 2)(\beta^2 + 2) = (\alpha\beta)^2 + 2(\alpha^2 + \beta^2) + 4$$

$$= \left(-\frac{1}{3}\right)^2 + 2\{(\alpha + \beta)^2 - 2\alpha\beta\} + 4$$

$$= \frac{1}{9} + 2\left\{\frac{1}{36} - 2 \times \left(-\frac{1}{3}\right)\right\} + 4 = \frac{198}{36}$$

$$\therefore \text{The required equation is } 36x^2 - 169x + 198 = 0$$

12. (c): Let the roots of the given equation be α, β

$$\Rightarrow \alpha + \beta = \omega \text{ and } \alpha\beta = v$$

Also given that $\alpha - \beta = 2$

$$\text{Now } (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow 4 = \omega^2 - 4v$$

$$\Rightarrow \omega^2 = 4(1 + v)$$

13. (c): From the given condition, we have

$$\alpha + \beta = -p, \alpha\beta = q$$

$$\text{Again } \alpha^4 + \beta^4 = r, \alpha^4\beta^4 = s$$

$$\therefore \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$$

$$= \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2 = (p^2 - 2q)^2 - 2q^2 \quad \dots(1)$$

$$\Rightarrow (p^2)^2 - 4q(p^2) + 2q^2 - r = 0 \quad \dots(2)$$

Let $p^2 = z$. Now, from (2) we get,

$$z^2 - 4qz + 2q^2 - r = 0 \quad \dots(3)$$

Thus, $p^2 = z$ is one root of the equation (3). Let the other root of the equation (3) be α .

Now for the equation (3)

$$D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r$$

$$= 8q^2 + 4[(p^2 - 2q)^2 - 2q^2]$$

$$= 8q^2 + 4(p^4 - 4qp^2 + 2q^2) = 4(p^2 - 2q)^2 > 0$$

$$\text{Now the equations } z^2 - 4qz + 2q^2 - r = 0$$

and $x^2 - 4qx + 2q^2 - r$ are identical

$$\Rightarrow D = 4(p^2 - 2q)^2 > 0$$

$\Rightarrow$ Roots are real and distinct.

$$14. (d): [x]^2 - 5[x] + 6 \leq 0$$

$$\Rightarrow ([x] - 2)([x] - 3) \leq 0$$

$$\Rightarrow 2 \leq [x] \leq 3$$

$$\Rightarrow 2 \leq [x] < 4$$

15. (d): Let the first term and common difference of the A.P. be S and T respectively.

$$\text{Therefore, } a = S + (p - 1)T, b = S + (q - 1)T$$

$$\text{and } c = S + (r - 1)T$$

Let us also suppose that the first term and then common ratio of the G.P. be respectively X and Y .

$$\text{Therefore } a = XY^{(p-1)},$$

$$b = XY^{(q-1)}$$

$$\text{and } c = XY^{(r-1)}$$

$$\text{Here the given equation is } (a^p b^q c^r) x^2 - (abc)x + (a^q b^p c^q) = 0$$

$\therefore$ Product of the roots of the given equation

$$= \frac{a^q b^p c^q}{a^p b^q c^r} = a^{q-p} \times b^{p-q} \times c^{q-r}$$

$$= (XY^{(p-1)})^{(q-p)} \times (XY^{(q-1)})^{(p-r)} \times (XY^{(r-1)})^{(q-p)}$$

$$= X^{(r-q+p-r+q-p)T} = X^{0T}$$

$$\times Y^{(p-1)(r-q) + (q-1)(p-r) + (r-1)(q-p)T}$$

$$= X^0 Y^0 = 1$$

16. (c) : The given equation is $y = \frac{mx^2 + 3x - 4}{m + 3x - 4x^2}$

$$\Rightarrow mx^2 + 3x - 4 = y(m + 3x - 4x^2)$$

$$\Rightarrow (4y + m)x^2 + 3(1 - y)x - (4 + my) = 0$$

Since, $x, y \in \mathbb{R}$, $\therefore D \geq 0$ gives

$$9(1 - y)^2 + 4(4y + m)(4 + my) \geq 0$$

$$\Rightarrow (9 + 16m)y^2 + (4m^2 + 46)m y + (9 + 16m) \geq 0$$

[If $ax^2 + bx + c \geq 0$ then $a > 0$ and $D \leq 0$]

$$\therefore 9 + 16m > 0 \text{ and } 4(2m^2 + 23)^2 - 4(9 + 16m)^2 \leq 0$$

$$\Rightarrow m > -\frac{9}{16} \text{ and } (2m^2 + 23)^2 - (9 + 16m)^2 \leq 0$$

$$\Rightarrow (2m^2 + 23 + 9 + 16m)(2m^2 + 23 - 9 - 16m) \leq 0$$

$$\Rightarrow 2(m^2 + 8m + 16) \times 2(m^2 - 8m + 7) \leq 0$$

$$\Rightarrow (m + 4)^2(m - 1)(m - 7) \leq 0$$

$$\Rightarrow 1 \leq m \leq 7 \text{ as } (m + 4)^2 \geq 0$$

17. (d) : Given that $\tan \theta + \tan 2\theta = -\tan 3\theta$

where $0 < \theta < \frac{\pi}{2}$.

$$\Rightarrow \tan \theta + \tan 2\theta + \tan 3\theta = 0$$

$$\Rightarrow \tan \theta + \tan 2\theta + \frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \tan 2\theta}$$

$$\Rightarrow (\tan \theta + \tan 2\theta) \left[1 + \frac{1}{1 - \tan \theta \tan 2\theta} \right] = 0$$

$$\therefore \tan \theta + \tan 2\theta = 0 \text{ or } (1 + 1 - \tan \theta \tan 2\theta) = 0$$

$$\Rightarrow \tan 2\theta = -\tan \theta \text{ or } \tan \theta \tan 2\theta = 2$$

$$\therefore 0 < \theta < \frac{\pi}{2} \therefore \tan \theta + \tan 2\theta \neq 0$$

$$\therefore \tan \theta \tan 2\theta = 2$$

18. (b) : Given that $a \tan \theta + b \sec \theta = c$

$$\Rightarrow c \cos \theta - a \sin \theta = b$$

$\therefore \alpha, \beta$ are the other two angles of the triangle,

So, $c \cos \alpha - a \sin \alpha = c \cos \beta - a \sin \beta$

$$\Rightarrow c(\cos \alpha - \cos \beta) = a(\sin \alpha - \sin \beta)$$

$$\Rightarrow 2c \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2} = 2a \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\Rightarrow \tan \frac{\alpha + \beta}{2} = -\frac{a}{c}$$

$$\text{Now, } \tan(\alpha + \beta) = 1 \left[\because \alpha + \beta = \frac{\pi}{4} \right]$$

$$\Rightarrow \frac{2 \tan \frac{\alpha + \beta}{2}}{1 - \tan^2 \frac{\alpha + \beta}{2}} = 1 \Rightarrow \frac{2 \left(-\frac{a}{c} \right)}{1 - \frac{a^2}{c^2}} = 1$$

$$\Rightarrow a^2 - c^2 = 2ac$$

19. (c) : We have, $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

$$\Rightarrow (1 - 3 \tan^2 \theta) \tan 3\theta = 3 \tan \theta - \tan^3 \theta$$

$$\Rightarrow (1 - 3 \tan^2 40^\circ) \tan 120^\circ = 3 \tan 40^\circ - \tan^3 40^\circ$$

$$\Rightarrow (1 - 3 \tan^2 40^\circ)^2 3 = (3 \tan 40^\circ - \tan^3 40^\circ)^2$$

[Squaring both sides]

$$\Rightarrow \tan^6 40^\circ - 33 \tan^4 40^\circ + 27 \tan^2 40^\circ = 3$$

20. (a) : Distance, d

$$= 12(\cos 36^\circ - \cos 72^\circ)$$

$$= 24 \sin 54^\circ \sin 18^\circ$$

$$= 24 \cos 36^\circ \sin 18^\circ$$

$$= 24 \cdot \frac{\sqrt{5} + 1}{4} \cdot \frac{\sqrt{5} - 1}{4} = 6 \text{ units}$$



21. (b): $f(x) = \sin^2 x + \sin^2 \left(x + \frac{\pi}{3} \right) + \cos x \cos \left(x + \frac{\pi}{3} \right)$

$$= \frac{1}{2} (1 - \cos 2x) + \frac{1}{2} \left[1 - \cos 2 \left(x + \frac{\pi}{3} \right) \right]$$

$$+ \frac{1}{2} \left[\cos \left(2x + \frac{\pi}{3} \right) + \cos \frac{\pi}{3} \right]$$

$$\text{Now, } f\left(\frac{\pi}{8}\right) = \frac{1}{2} \left(1 - \frac{1}{\sqrt{2}} \right) + \frac{1}{2} \left(1 + \frac{\sqrt{3} + 1}{2\sqrt{2}} \right) + \frac{1}{2} \left[\frac{-\sqrt{3} + 1}{2\sqrt{2}} + \frac{1}{2} \right]$$

$$= \frac{1}{2} \left[1 - \frac{1}{\sqrt{2}} + 1 + \frac{\sqrt{3} + 1}{2\sqrt{2}} - \frac{\sqrt{3} + 1}{2\sqrt{2}} + \frac{1}{2} \right] = \frac{1}{2} \left(\frac{5}{2} \right) = \frac{5}{4} \forall x$$

$$\text{Therefore, } gf\left(\frac{\pi}{8}\right) = g\left(\frac{5}{4}\right) = 1$$

22. (c) : $\cos^2 A + \cos^2 (B - A) - 2 \cos A \cos B \cos (A - B)$

$$= \cos^2 A + \cos^2 (A - B) [\cos (A - B) - 2 \cos A \cos B]$$

$$= \cos^2 A + \cos (A - B)$$

$$[[\cos (A - B) - \cos (A + B) - \cos (A - B)]]$$

$$= \cos^2 A - \cos (A - B) \cos (A + B)$$

$$= \cos^2 A - \cos^2 A + \sin^2 B = \sin^2 B$$

$$= \sin^2 B$$

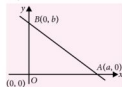
23. (b) : The equation of the straight line passing through the point of intersection of the lines $x + 2y = 1$ and $2x - y = 1$ is

$$(x + 2y - 1) + \lambda(2x - y - 1) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (2 - \lambda)y = (1 + \lambda)$$

$$\Rightarrow \frac{x}{\frac{1 + \lambda}{1 + 2\lambda}} + \frac{y}{\frac{2 - \lambda}{2 - \lambda}} = 1$$

$$\therefore A = \left(\frac{1 + \lambda}{1 + 2\lambda}, 0 \right)$$



$$\text{and } B \equiv \left(0, \frac{1+\lambda}{2-\lambda}\right)$$

Let the mid-point of AB be (h, k)

$$\therefore h = \frac{1}{2} \left(\frac{1+\lambda}{1+2\lambda} \right) \text{ and } k = \frac{1}{2} \left(\frac{1+\lambda}{2-\lambda} \right)$$

Eliminating λ from h and k , we get $h + 3k = 10hk$.

Therefore the required locus is $x + 3y = 10xy$.

24. (a) : Given that $2x + y - 1 = 0$, $ax + 3y - 3 = 0$ and $3x + 2y - 2 = 0$ are concurrent. Therefore all the given line must pass through the same point. Now the point of intersection of the lines $2x + y - 1 = 0$ and $3x + 2y - 2 = 0$ is $(0, 1)$. Clearly the line $ax + 3y = 0$ is passing through the point $(0, 1)$ for all real values of a .

25. (b) **26. (c)**

27. (d) : The given equations of the straight line are

$$\frac{x}{\cos \theta} + \frac{y}{\sin \theta} = a \text{ and } \frac{x}{\sec \theta} - \frac{y}{\operatorname{cosec} \theta} = \frac{a}{\sec 2\theta}$$

$$\therefore R_1 = \frac{-a}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \text{ and } R_2 = \frac{-a \cos 2\theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}}$$

$$\Rightarrow R_1^2 = \frac{a^2}{\sec^2 \theta + \operatorname{cosec}^2 \theta} = a^2 \frac{1}{4} \sin^2 2\theta$$

$$\text{and } R_2^2 = \frac{a^2 \cos^2 2\theta}{\cos^2 \theta + \sin^2 \theta} = a^2 \cos^2 2\theta$$

$$\therefore 4R_1^2 + R_2^2 = a^2 (\sin^2 2\theta + \cos^2 2\theta) = a^2$$

28. (b) : Let the required equation is $\frac{x}{a} + \frac{y}{b} = 1$

$$\text{Clearly, } -5 = \frac{2a}{3} \text{ and } 4 = \frac{b}{3}$$

$$\Rightarrow a = -\frac{15}{2} \text{ and } b = 12$$

$$\therefore \text{The required equation is } \frac{x}{-15} + \frac{y}{12} = 1$$

29. (b) : The equation of the line is $3x - 4y = 56 \dots (1)$ and that of the curve is

$$x^2 + y^2 - 24x + 10y = 0 \dots (2)$$

Now homogenizing the equation (1) and (2) we get

$$x^2 + y^2 - (24x - 10y) \left(\frac{3x - 4y}{56} \right) = 0$$

$$\Rightarrow 56(x^2 + y^2) - (72x^2 - 126xy + 40y^2) = 0$$

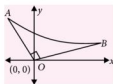
$$\Rightarrow 8x^2 - 63xy - 8y^2 = 0$$

$$\Rightarrow (8x + y)(x - 8y) = 0$$

$$\therefore y = -8x \text{ and } x = 8y$$

which are the equations of

line OA and OB .



Clearly they are perpendicular to each other.

30. (b) : The nearest point on the lines $3x + 4y = 25 \dots (1)$ from the origin is the foot of the perpendicular from the origin to the line (1).

Now, let the perpendicular to equation (1) is

$$4x - 3y + k = 0 \dots (2)$$

Since (2) is passing through $(0, 0) \therefore k = 0$

$$\text{So (2) becomes } 4x - 3y = 0 \dots (3)$$

Now solving (1) and (3), we get the required point, which is $(3, 4)$.

CBSE Class 10 result 2018: Nearly two lakh students scored over 90% in Maths

Nearly two lakh or 12.5% students scored 90% and above in the Central Board of Secondary Education (CBSE) class 10 Mathematics. The 2017-18 batch of students are the first batch to attempt the reintroduced CBSE compulsory class 10 exams. The board had replaced the compulsory exams with a Continuous Comprehensive Evaluation (CCE) system of assessment in 2009 and had made the class 10 exam optional.

Students had the option to either attempting the school-based or board-based final exam. Instead of marks, students were given grades. According to data shared by CBSE, 1,99,884 students scored 90 and above marks in the Mathematics paper. In 2017, when students used to get grades, 1,89,890 students had scored A1 grade, which is equivalent to 90 and above. "The number of students with 100 marks

Number of students with 100 marks

Hindi Course A: 244

Mathematics: 3,131

Hindi Course B: 643

Science: 2,244

Social Science: 6,469

English: 684

in Mathematics is 3,131. The highest number of perfect scorers is for Social Sciences, in which 6,469 students secured 100 marks," a CBSE official said.

MATH archives



Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. The centre of a square $ABCD$ is at z_0 . If A is z_1 , then the centroid of the $\triangle ABC$ is

(a) $2z_0 - i(z_1 - z_0)$ (b) $z_0 + i\left(\frac{z_1 - z_0}{3}\right)$
 (c) $\frac{z_0 + iz_1}{3}$ (d) $\frac{2}{3}(z_1 - z_0)$

2. The reflection of the complex number $(2 - i)$ in the straight line $iz = \bar{z}$ is

(a) $4 - 3i$ (b) $3 + 4i$ (c) $2 + i$ (d) $1 - 2i$

3. If sum of the arguments of all the roots of the equation $z^n = k$ (k is real) is odd multiple of π , then it can be true when

- (i) n is odd and $k > 0$
 (ii) n is odd and $k < 0$
 (iii) n is even and $k > 0$
 (iv) n is even and $k < 0$
 (a) (i) and (iii) are correct
 (b) (i) and (iv) are correct
 (c) (ii) and (iii) are correct
 (d) (ii) and (iv) are correct

4. Let $f: R \rightarrow \left(0, \frac{\pi}{2}\right)$ be a function defined by $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$, then complete set of values of α for which $f(x)$ is onto, is

(a) $\left[\frac{1 - \sqrt{17}}{2}, \frac{1 + \sqrt{17}}{2}\right]$
 (b) $\frac{1 \pm \sqrt{17}}{2}$

(c) $\left(-\infty, \frac{1 - \sqrt{17}}{2}\right] \cup \left[\frac{1 + \sqrt{17}}{2}, \infty\right)$

- (d) none of these

5. Let $g(x)$ and $f(x)$ be twice differentiable functions in R and $f(2) = 8$, $g(2) = 0$, $f(4) = 10$ and $g(4) = 8$, then

- (a) $g'(x) > 4f'(x) \forall x \in (2, 4)$
 (b) $g(x) > f(x) \forall x \in (2, 4)$
 (c) $3g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$
 (d) $g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$

6. If A, G, H be respectively the AM, GM and the HM between two positive numbers and if $xA = yG = zH$ where x, y, z are non zero positive quantities, then x, y, z are in

- (a) A.P. (b) G.P. (c) H.P. (d) A.G.P

7. Given $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = \ln 2$, then the value of

the definite integral $\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x} dx$ is

- (a) $\frac{1}{2} \ln 2$ (b) $\frac{\pi}{2} - \ln 2$
 (c) $\frac{\pi}{4} - \frac{1}{2} \ln 2$ (d) $\frac{\pi}{2} + \ln 2$

8. Let $y = \{x\}^{[x]}$ where $\{x\}$ denotes the fractional part of x and $[x]$ denotes greatest integer $\leq x$, then $\int_0^3 y dx =$

- (a) $\frac{11}{6}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{5}{6}$

9. A quadratic equation $fx = ax^2 + bx + c = 0$ ($a \neq 0$) has positive distinct roots reciprocal of each other. Which one is correct?

- (a) $f'(1) = 0$ (b) $af'(1) < 0$
(c) $af'(1) > 0$
(d) Nothing can be said about $af'(1)$

10. Each of 10 passengers board any of the three buses randomly, which had no passenger initially. The probability that each bus has got at least one passenger, is

- (a) $\frac{10P_3 \cdot 3^7}{3^{10}}$ (b) $1 - \frac{10C_3 \cdot 3^7}{3^{10}}$
(c) $1 - \frac{2^{10}}{3^{10}}$ (d) $\frac{3^{10} - 3 \cdot 2^{10} + 3}{3^{10}}$

SOLUTIONS

1. (b): From the figure,

$$z_2 - z_0 = i(z_1 - z_0)$$

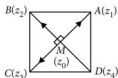
$$\Rightarrow z_2 = z_0 + i(z_1 - z_0)$$

$$\text{and } z_3 - z_0 = -(z_1 - z_0)$$

$$\Rightarrow z_3 = 2z_0 - z_1$$

Hence, centroid of $\triangle ABC$

$$= \frac{z_1 + z_2 + z_3}{3} = z_0 + i\left(\frac{z_1 - z_0}{3}\right)$$



2. (d): $iz = \bar{z}$

$$\Rightarrow i(x + iy) = x - iy \Rightarrow i(x + y) - (x + y) = 0$$

$$\Rightarrow (i - 1)(x + y) = 0$$

which represents the line $y = -x$

So, reflection of the point $(2, -1)$ in the line $y = -x$ gives the point $(1, -2)$.

3. (c): When n is odd, and $k > 0$

Sum of arguments = even multiple of π .

When n is odd and $k < 0$

Sum of arguments = odd multiple of π .

When n is even and $k > 0$

Sum of arguments = odd multiple of π .

When n is even and $k < 0$

Sum of arguments = even multiple of π .

4. (b): Clearly $x^2 + 4x + \alpha^2 - \alpha \geq 0 \quad \forall x \in R$ and must take all values of the interval $[0, \infty)$

$$\Rightarrow D = 0$$

$$\text{i.e., } 16 - 4(\alpha^2 - \alpha) = 0 \Rightarrow \alpha^2 - \alpha = 4$$

$$\Rightarrow \alpha = \frac{1 \pm \sqrt{17}}{2}$$

5. (d): Let $h(x) = g(x) - 4f(x)$

Verify the Rolle's theorem in $(2, 4)$

$$\text{Now } h(2) = g(2) - 4f(2) = 0 - 4 \times 8 = -32$$

$$h(4) = g(4) - 4f(4) = 8 - 40 = -32$$

$$\Rightarrow h'(x) = 0 \text{ for at least one } x \in (2, 4)$$

$$\therefore g'(x) = 4f'(x) \text{ for at least one } x \in (2, 4).$$

6. (b): Let two numbers be a and b

$$x \cdot \frac{a+b}{2} = y \cdot \sqrt{ab} = z \cdot \frac{2ab}{a+b} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{y}{x} = \frac{z}{y}$$

$$\Rightarrow y^2 = xz \Rightarrow x, y, z \text{ are in GP.}$$

$$7. (c): I = \int_0^{\pi/2} \frac{1 + \sin x + \cos x - 1 - \cos x}{1 + \sin x + \cos x} dx$$

$$= \int_0^{\pi/2} dx - \int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} - \int_0^{\pi/2} \frac{\cos x}{1 + \sin x + \cos x} dx$$

$$\therefore I = \frac{\pi}{4} - \frac{1}{2} \ln 2$$

$$8. (a): \int_0^3 (x - [x])^{[x]} dx$$

$$= \int_0^1 dx + \int_1^2 (x-1) dx + \int_2^3 (x-2)^2 dx$$

$$\text{Put } y = x - 1, \quad z = x - 2$$

$$= 1 + \int_0^1 y dy + \int_0^1 z^2 dz = 1 + \left[\frac{y^2}{2} \right]_0^1 + \left[\frac{z^3}{3} \right]_0^1$$

$$= 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

9. (b): $f(x) = ax^2 + bx + c = 0$ ($a \neq 0$)

Let α and $\frac{1}{\alpha}$ be its roots.

$$\therefore \alpha + \frac{1}{\alpha} = -\frac{b}{a} > 2 \Rightarrow \left(\frac{2a+b}{a} \right) < 0 \Rightarrow a(2a+b) < 0$$

$$f'(x) = 2ax + b$$

$$af'(1) = a(2a+b) < 0$$

10. (d): Total ways in which they can be seated in the buses = 3^{10}

Number of cases in which each bus has got at least one passenger

$$= 3^{10} - {}^3C_1 \times 1 - {}^3C_2(2^{10} - 2)$$

$\therefore$ Required probability

$$= \frac{3^{10} - {}^3C_1 \times 1 - {}^3C_2(2^{10} - 2)}{3^{10}} = \frac{3^{10} - 3 \cdot 2^{10} + 3}{3^{10}}$$

Challenging PROBLEMS



Binomial Theorem



1. For $n \in N$, the product

$$\left(4 - \frac{2}{1}\right) \left(4 - \frac{2}{2}\right) \left(4 - \frac{2}{3}\right) \left(4 - \frac{2}{4}\right) \dots \left(4 - \frac{2}{n}\right) \text{ is}$$

- (a) an integer for all n
 (b) a rational number for all n
 (c) an integer only for $n \leq 16$
 (d) an irrational number

2. Let $f(n) = \sum_{k=1}^n k^2 \cdot {}^nC_k^2$ then $f(5) =$

- (a) 1000 (b) 1250
 (c) 1750 (d) 2500

3. If $\sum_{r=0}^n \left(\frac{r^3 + 2r^2 + 3r + 2}{(r+1)^2} \right) {}^nC_r = \frac{2^2 + 2^3 + 2^4 - 2^1}{3}$,

then $n =$

- (a) 2 (b) 4 (c) 8 (d) 16

4. Let $T = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ and

$$Y = {}^nC_1 - \frac{1}{2} {}^nC_2 + \frac{1}{3} {}^nC_3 \dots (n \text{ terms}), \text{ then}$$

- (a) $T = \frac{1}{2} Y$ (b) $T = Y$
 (c) $T = 2Y$ (d) $T = \frac{Y}{3}$

5. $\sum_{k=0}^n \frac{{}^nC_k}{(k+1)(k+2)} =$

- (a) $\frac{2^{n+2} - (n+3)}{(n+1)(n+2)}$ (b) $\frac{2^{n+3} - (n+2)}{(n+1)(n+2)}$
 (c) $\frac{2^n - (n+2)}{n(n+1)}$ (d) $\frac{2^n - (n+3)}{n(n+2)}$

6. Let n be an odd integer greater than or equal to 5, then ${}^nC_1 - 5 \cdot {}^nC_2 + 5^2 \cdot {}^nC_3 - \dots + 5^{n-1} \cdot {}^nC_n$ is

- (a) a prime number (b) a composite number
 (c) prime or composite depends on value of n
 (d) a prime number only when $n > 25$

7. Let n be an odd positive integer. Consider the set

$$S = \left\{ {}^nC_1, {}^nC_2, \dots, {}^nC_{\frac{n-1}{2}} \right\}, \text{ then}$$

- (a) S contains an odd number of odd terms
 (b) S contains an even number of odd terms
 (c) S contains an odd number of even terms
 (d) S contains an even number of even terms

8. For positive integers n and k , we have

$$\frac{({}^{n-1}C_{n-1})^6 + ({}^{n-2}C_k)^6 + ({}^{n+3}C_{n+1})^3}{3({}^{n-2}C_k)^2({}^{n+3}C_2)} = n^2 \text{ then } n + k =$$

- (a) 8 (b) 0 (c) 16 (d) 60

9. Evaluate: $17 \cdot {}^{20}C_0 + 16 \cdot {}^{20}C_1 + 15 \cdot {}^{20}C_2 + \dots$
 $+ 0 \cdot {}^{20}C_{17} + (-1) \cdot {}^{20}C_{18} + (-2) \cdot {}^{20}C_{19} + (-3) \cdot {}^{20}C_{20}$

- (a) $7 \cdot 2^{20}$ (b) $7 \cdot 2^{19}$
 (c) $14 \cdot 2^{20}$ (d) $14 \cdot 2^{19}$

10. $\sum_{i=0}^{[n/2]} {}^nC_i = \text{---}$, $[\cdot]$ denotes greatest integer function

- (a) 2^n if n is odd (b) 2^{n+1} if n is odd

- (c) $2^{n-1} + \frac{1}{2} \cdot {}^nC_{n/2}$ if n is even

- (d) $2^n + \frac{1}{2} \cdot {}^nC_{n/2}$ if n is even

11. $\sum_{k=0}^{49} (-1)^k \cdot ({}^{99}C_{2k}) =$

- (a) 2^{49} (b) -2^{49} (c) 2^{50} (d) -2^{50}

12. All solutions of (n, k) ($2 < k < n$) such that ${}^nC_{k-1}, {}^nC_k$ and ${}^nC_{k+1}$ form an increasing A.P. is of the form ($t \geq 3$)

- (a) $\left[t^2 - 2, \frac{t^2 - t - 2}{2}\right]$ (b) $\left[t^2 - 2, \frac{t^2 - t}{2}\right]$
 (c) $[t^2 - 2, t^2]$ (d) $\left[t^2 - 2, \frac{t^2 - t + 2}{2}\right]$

13. Let sequence, $a_n = {}^{n+4}C_4 - a_{n-2} + 2a_{n-1}$, $n \geq 2$, $a_0 = 0$, $a_1 = 5$ then $a_n =$

- (a) ${}^{n+6}C_6 - n - 1$ (b) ${}^{n+6}C_6 - n - 2$
 (c) ${}^{n+6}C_6 - n + 1$ (d) ${}^{n+6}C_6 - n + 2$

14. For a non-negative integer k , define

$$T_k(n) = 1^k + 2^k + \dots + n^k \text{ then } 1 + \sum_{k=0}^{r-1} {}^rC_k T_k(n) =$$

- (a) n^r (b) $(n+1)^r$
 (c) $n^r + 1$ (d) $n^r - r$

15. Let m and n be integers such that $1 \leq m \leq n$, then

the number $n \sum_{k=0}^{m-1} (-1)^k \cdot {}^nC_k$ is divisible by

- (a) m (b) $m + 1$
 (c) $m + 2$ (d) $m + 3$

16. Let $\langle F_n \rangle$ be the fibonacci sequence, $F_1 = F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$, then for any positive integer n ,

$$F_1({}^nC_1) + F_2({}^nC_2) + \dots + F_n({}^nC_n) =$$

- (a) F_{2n} (b) F_{2n+1} (c) F_{2n+2} (d) F_{2n-1}

Comprehension

For a positive integer n , define the integers a_n, b_n and c_n by $a_n + b_n \cdot \sqrt[3]{2} + c_n \cdot \sqrt[3]{4} = (1 + \sqrt[3]{2} + \sqrt[3]{4})^n$

17. If $n = 3m$ form then $2^{-n/3} \cdot \sum_{k=0}^n ({}^nC_k) \cdot a_k =$

- (a) a_n (b) a_{n+1}
 (c) $b_n \cdot \sqrt[3]{2}$ (d) c_n

18. If $n = 3m + 2$ form then $2^{-n/3} \cdot \sum_{k=0}^n {}^nC_k \cdot a_k =$

- (a) a_n (b) a_{n+1}
 (c) $b_n \cdot \sqrt[3]{2}$ (d) $c_n \cdot \sqrt[3]{4}$

19. If $n = 3m + 1$ form then $2^{-n/3} \cdot \sum_{k=0}^n {}^nC_k \cdot a_k =$

- (a) a_n (b) a_{n+1}
 (c) b_n (d) $c_n \cdot \sqrt[3]{4}$

SOLUTIONS

$$1. (a) : \prod_{k=1}^n \left(4 - \frac{2}{k}\right) = 2^n \cdot \frac{\prod (2k-1)}{n!} \\ = \frac{2^n \cdot \prod (2k-1) \cdot \prod (2k)}{n! \cdot \prod (2k)} = \frac{2^n \cdot (2n)!}{2^n \cdot (n!)^2} = {}^{2n}C_n = \text{integer}$$

2. (c) : Since, $k \cdot {}^nC_k = n \cdot {}^{n-1}C_{k-1}$

$$\therefore \sum_{k=1}^n k^2 \cdot ({}^nC_k)^2 = n^2 \cdot \sum_{k=0}^{n-1} ({}^{n-1}C_k)^2 \\ = n^2 \cdot ({}^{2n-2}C_{n-1})$$

$$\text{So, } f(5) = 25 \cdot {}^8C_4 = 1750$$

$$3. (a) : \text{Given summation} = \sum_{r=0}^n \frac{(r^2 + r + 2) \cdot {}^nC_r}{r + 1} \\ = \sum_{r=0}^n r \cdot {}^nC_r + 2 \sum_{r=0}^n \frac{{}^nC_r}{r + 1} = n \cdot 2^{n-1} + 2 \frac{[2^{n+1} - 1]}{n + 1}$$

So, by comparing with expression on R.H.S., $n = 2$.

$$4. (b) : \text{Notice that, } \frac{1 - (1-x)^n}{x} \\ = {}^nC_1 - {}^nC_2 x + {}^nC_3 x^2 - \dots$$

Integrating on both sides from 0 to 1, we get

$$\int_0^1 \frac{1-y^n}{1-y} dy = {}^nC_1 - {}^nC_2 \cdot \frac{1}{2} + {}^nC_3 \cdot \frac{1}{3} \dots \\ \int_0^1 (1+y+y^2+\dots+y^{n-1}) dy = {}^nC_1 - \frac{1}{2} {}^nC_2 + \frac{1}{3} {}^nC_3 \dots$$

$$\text{or } 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = {}^nC_1 - \frac{1}{2} \cdot {}^nC_2 + \frac{1}{3} \cdot {}^nC_3 + \dots (n \text{ terms})$$

5. (a) : Summing of the identity

$$n+2 {}^nC_k = \frac{(n+2)(n+1)}{(n+2-k)(n+1-k)} \cdot {}^nC_k$$

from $k = 0$ to $k = n$, gives

$$S_n = \frac{1}{(n+2)(n+1)} \left(\sum_{k=0}^{n+2} {}^nC_k - {}^{n+2}C_{n+1} - {}^{n+2}C_{n+2} \right)$$

$$\text{So, } S_n = \frac{1}{(n+2)(n+1)} [2^{n+2} - (n+2) - 1] = \frac{2^{n+2} - (n+3)}{(n+1)(n+2)}$$

6. (b) : Let $N = {}^nC_1 - 5 \cdot {}^nC_2 + 5^2 \cdot {}^nC_3 - \dots + 5^{n-1} \cdot {}^nC_n$

$$\text{then } 5N = 1 + (-1 + 5)^n$$

So,

$$N = \frac{4^n + 1}{5} = \frac{1}{5} \left[2^{n-1} + \left(2^{\frac{n-1}{2}} - 1 \right)^2 \right] \left[2^{n-1} + \left(2^{\frac{n-1}{2}} + 1 \right)^2 \right]$$

Notice that for $n \geq 5$, both the factors are greater than 5 and one of the factors is divisible by 5. Hence, N is not a prime number.

7. (a) : Define $S_n = {}^nC_1 + {}^nC_2 + \dots + \frac{{}^nC_{n-1}}{2}$, then

$$2S_n = {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} = 2^n - 2$$

$$\text{i.e. } S_n = 2^{n-1} - 1 = \text{odd}$$

$\Rightarrow S$ contains an odd number of odd terms.

8. (a) : Applying A.M.-G.M. inequality on $({}^nC_{n-1})^6$, $({}^{n-2}C_k)^6$ and $({}^{n+3}C_{n+1})^3$. We have A.M. = G.M. from the given condition in question.

So, all the given numbers should be same.

$$\text{i.e. } ({}^nC_{n-1})^6 = ({}^{n-2}C_k)^6 = ({}^{n+3}C_{n+1})^3$$

$$\text{i.e. } n^2 = ({}^{n-2}C_k)^2 = ({}^{n+3}C_2)$$

Hence, $n = 6$ and $k = 2$

9. (a) : Given sum = $\sum_{k=0}^{20} (17-k) \cdot {}^{20}C_k$

$$= 17 \sum_{k=0}^{20} {}^{20}C_k - \sum_{k=0}^{20} k \cdot {}^{20}C_k$$

$$= 17(2^{20}) - 20(2^{19}) = 7(2^{20})$$

10. (c) : For $n = 2m + 1$ form (odd)

$$S_n = {}^{2m+1}C_0 + {}^{2m+1}C_1 + \dots + {}^{2m+1}C_m$$

$$\Rightarrow 2S_n = {}^{2m+1}C_0 + {}^{2m+1}C_1 + \dots + {}^{2m+1}C_m + {}^{2m+1}C_{m+1} + \dots + {}^{2m+1}C_{2m+1}$$

$$= 2^{2m+1} \quad \text{i.e. } S_n = 2^{2m}$$

Similarly for $n = 2m$ (even), $2S_n = 2^{2m} + 2^m C_m$

$$\text{i.e. } S_n = 2^{n-1} + \frac{1}{2} \cdot {}^nC_{n/2}$$

$$11. (b) : (1+x)^{99} = {}^{99}C_0 + {}^{99}C_1 x + {}^{99}C_2 x^2 + \dots + {}^{99}C_{99} x^{99}$$

We put $x = i$ and $x = -i$ and add them to get

$${}^{99}C_0 - {}^{99}C_2 + {}^{99}C_4 - \dots = \frac{(1+i)^{99} + (1-i)^{99}}{2} = -2^{49}$$

Noting that $(1+i)^2 = 2i$ makes simplification easier.

12. (a) : For A.P., $2 \cdot {}^nC_k = {}^nC_{k-1} + {}^nC_{k+1}$ gives

$$(n-2k)^2 = n+2 \quad \text{i.e. if } n-2k = t \text{ then}$$

$$n = t^2 - 2 \text{ and } k = \frac{t^2 - t - 2}{2}, t \geq 3 \text{ as } n > 2.$$

13. (a) : Let $a_n - a_{n-1} = b_n$, $b_1 = 5$ then given relation becomes $b_n - b_{n-1} = {}^{n+4}C_4$, $n \geq 2$

Putting $n = 2, 3, \dots$ and adding all, we have

$$b_n - b_1 = \sum_{r=2}^n {}^{r+4}C_4 = {}^{n+5}C_5 - 6$$

$$\text{i.e. } a_n - a_{n-1} = {}^{n+5}C_5 - 1$$

Putting $n = 1, 2, 3, \dots$ and adding all, we have

$$a_n = \sum_{r=1}^n {}^{r+5}C_5 - n = {}^{n+6}C_6 - 1 - n$$

$$14. (b) : 1 + \sum_{k=0}^{r-1} {}^rC_k \cdot T_k(n) = 1 + \sum_{k=0}^{r-1} {}^rC_k \cdot \sum_{p=1}^n p^k$$

$$= 1 + \sum_{p=1}^n \sum_{k=0}^{r-1} {}^rC_k \cdot p^k = 1 + \sum_{p=1}^n [(p+1)^r - p^r]$$

$$= (n+1)^r$$

15. (a) : We have, $n \cdot \sum_{k=0}^{m-1} (-1)^k \cdot {}^nC_k$

$$= n \cdot \sum_{k=0}^{m-1} (-1)^k \cdot [{}^{n-1}C_k + {}^{n-1}C_{k-1}]$$

$$= n \cdot \sum_{k=0}^{m-1} (-1)^k \cdot {}^{n-1}C_k - n \sum_{k=0}^{m-2} (-1)^k \cdot {}^{n-1}C_k$$

$$= n \cdot (-1)^{m-1} \cdot ({}^{n-1}C_{m-1}) = m(-1)^{m-1} \cdot {}^nC_m$$

Which is divisible by m .

16. (a) : The general term in fibonacci sequence is given by

$$F_n = \frac{1}{\sqrt{5}} (\alpha^n - \beta^n), \quad n \geq 1 \text{ where } \alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$$

$$\text{So, } \sum_{i=1}^n F_i ({}^nC_i) = \frac{1}{\sqrt{5}} \sum_{i=1}^n [{}^nC_i \alpha^i - {}^nC_i \beta^i]$$

$$= \frac{1}{\sqrt{5}} [(1+\alpha)^n - (1+\beta)^n] = \frac{1}{\sqrt{5}} [\alpha^{2n} - \beta^{2n}]$$

$$= F_{2n} [\because 1 + \alpha = \alpha^2]$$

(17-19):

$$17. (a) \quad 18. (c) \quad 19. (d)$$

$$a_n + b_n \cdot \sqrt[3]{2} + c_n \cdot \sqrt[3]{4} = \frac{(\sqrt[3]{2})^n (1 + \sqrt[3]{2} + \sqrt[3]{4})^n}{(\sqrt[3]{2})^n}$$

$$= 2^{-n/3} [\sqrt[3]{2} + \sqrt[3]{4} + 2]^n = 2^{-n/3} [1 + (1 + \sqrt[3]{2} + \sqrt[3]{4})^n]$$

$$= 2^{-n/3} \cdot \sum_{k=0}^n {}^nC_k (a_k + b_k \cdot \sqrt[3]{2} + c_k \cdot \sqrt[3]{4})$$

$$\text{Hence, } a_n + b_n \cdot \sqrt[3]{2} + c_n \cdot \sqrt[3]{4} = 2^{-n/3} \sum_{k=0}^n {}^nC_k \cdot a_k$$

$$+ 2^{-n/3} \sum_{k=0}^n {}^nC_k \cdot b_k \cdot \sqrt[3]{2} + 2^{-n/3} \sum_{k=0}^n {}^nC_k \cdot c_k \cdot \sqrt[3]{4}$$



OLYMPIAD CORNER



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1. Evaluate the product $\prod_{k=0}^{1999} \left(4 \sin^2 \frac{k\pi}{2^{2000}} - 3 \right)$
2. Let $a, b, c \in \mathbb{Q}$, $ac \neq 0$. Given that the equation $ax^2 + bxy + cy^2 = 0$ has a nonzero solution of the form $(x, y) = (a_0 + a_1 \sqrt[3]{2} + a_2 \sqrt[3]{4}, b_0 + b_1 \sqrt[3]{2} + b_2 \sqrt[3]{4})$

with $a_i, b_i \in \mathbb{Q}$, $i = 0, 1, 2$, prove that it has also has a nonzero rational solution.

3. Does there exist a bijection f of
 - (a) a plane with itself
 - (b) three-dimensional space with itself such that for any distinct points A, B line AB and line $f(A)f(B)$ are perpendicular?

4. Given a triangle ABC , show how to construct, with straight, edge and compass, a triangle $A'B'C'$ with minimal area such that A', B', C' lie on AB, BC, CA , respectively, $\angle B'A'C' = \angle BAC$, and $\angle A'C'B' = \angle ACB$.

5. Find all triples (x, y, z) of natural numbers such that y is a prime number, y and 3 do not divide z , and $x^3 - y^3 = z^2$.

6. Let B_1 and C_1 be points on the sides AC and AB of triangle ABC . Lines BB_1 and CC_1 intersect at point D . Prove that a circle can be inscribed inside quadrilateral AB_1DC_1 if and only if the incircles of the triangles ABD and ACD are tangent to each other.

7. Find the number of all natural numbers n , $4 \leq n \leq 1023$, whose binary representations do not contain three consecutive equal digits.

8. Let x, y, z be non negative real numbers such that $x + y + z = 1$. Prove that $x^2y + y^2z + z^2x \leq \frac{4}{27}$, and determine when equality occurs.

9. For which acute-angled triangle is the ratio of the shortest side to the inradius maximal?

10. Determine the maximum value of λ such that if $f(x) = x^3 + ax^2 + bx + c$ is a cubic polynomial with all its roots nonnegative, then $f(x) \geq \lambda(x - a)^3$ for all $x \geq 0$. Find the equality condition.

11. Show that there exists a triangle ABC for which, with the usual labelling of sides and medians, it is true that $a \neq b$ and $a + m_a = b + m_b$. Show further that there exists a number k such that for each such triangle $a + m_a = b + m_b = k(a + b)$. Finally, find all possible ratios $a : b$ of the sides of these triangles.

12. For any given positive integer n , determine (as a function of n) the number of ordered pairs (x, y) of positive integers such that $x^2 - y^2 = 10^2 \cdot 30^{2n}$. Prove further that the number of such pairs is never a perfect square.

13. Find all nonzero reals $x > -1$ which satisfy

$$\frac{x^2}{(x+1-\sqrt{x+1})^2} < \frac{x^2+3x+18}{(x+1)^2}.$$

14. Let D, E, F be points on the sides BC, CA, AB , respectively, of triangle ABC such that $AD \perp BC$, $AF = FB$, and BE is the angle bisector of $\angle B$. Prove that AD, BE, CF are concurrent if and only if, $a^2(a - c) = (b^2 - c^2)(a + c)$, where $a = BC, b = CA, c = AB$.

15. Suppose that for any real x with $|x| \neq 1$, a function $f(x)$ satisfies $f\left(\frac{x-3}{x+1}\right) + f\left(\frac{3+x}{1-x}\right) = x$.

Find all possible $f(x)$.

16. (a) Let $a, b \in \mathbb{R}$, $a < b$. Prove that $a < x < b$ if and only if there exists $0 < \lambda < 1$ such that $x = \lambda a + (1 - \lambda)b$.

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(b) The function $f: R \rightarrow R$ has the property:

$$f(\lambda x + (1 - \lambda)y) < \lambda f(x) + (1 - \lambda)f(y)$$

for all $x, y \in R$, and all $0 < \lambda < 1$. Prove that one cannot find four points on the function's graph that are the vertices of a parallelogram.

SOLUTIONS

1. For all real x , let $f(x) = \sin\left(\frac{x\pi}{2^{2000}}\right)$

When $k = 0$, the expression inside the parentheses equals -3 . Using the triple angle formula $\sin(3\theta) = 4\sin^3\theta - 3\sin\theta$, and noting that $f(k) \neq 0$ when $1 \leq k \leq 2^{1999}$, we can rewrite the given product as

$$-3 \prod_{k=1}^{2^{1999}} \frac{\sin\left(\frac{3k\pi}{2^{2000}}\right)}{\sin\left(\frac{k\pi}{2^{2000}}\right)} \text{ or } -3 \prod_{k=1}^{2^{1999}} \frac{f(3k)}{f(k)}. \quad \dots(1)$$

Observe that

$$\prod_{k=1}^{2^{1999}} f(3k) = \prod_{k=1}^3 f(3k) \cdot \prod_{k=\frac{2^{1999}+1}{3}}^{\frac{2^{2000}-1}{3}} f(3k) \cdot \prod_{k=\frac{2^{2000}+2}{3}}^{2^{1999}} f(3k)$$

Because $\sin\theta = \sin(\pi - \theta) = -\sin(\pi + \theta)$, we have

$$f(x) = f(2^{2000} - x) = -f(x - 2^{2000})$$

Hence, letting $S_i = \{k, | 1 \leq k \leq 2^{1999}, k \equiv i \pmod{3}\}$ for $i = 0, 1, 2$, the last expression equals

$$\begin{aligned} & \prod_{k=1}^{\frac{2^{1999}-2}{3}} f(3k) \prod_{k=\frac{2^{2000}-1}{3}}^{\frac{2^{2000}-1}{3}} f(2^{2000} - 3k) \cdot \prod_{k=\frac{2^{2000}+2}{3}}^{2^{1999}} (-f(3k - 2^{2000})) \\ &= \prod_{k \in S_0} f(k) \prod_{k \in S_1} f(k) \prod_{k \in S_2} (-f(k)) \\ &= (-1)^{\frac{2^{1999}+1}{3}} \prod_{k=1}^{2^{1999}} f(k) = - \prod_{k=1}^{2^{1999}} f(k) \end{aligned}$$

Combined with the expression in (1), this implies that the desired product is $(-3)(-1) = 3$.

2. Let $(\alpha, \beta) = (a_0 + a_1\sqrt[3]{2} + a_2\sqrt[3]{4}, b_0 + b_1\sqrt[3]{2} + b_2\sqrt[3]{4})$

be the given solution. If $\beta = 0$ then $\alpha\alpha^2 = 0$, which is impossible because a and α must both be nonzero.

Therefore, $\beta \neq 0$, and $\frac{\alpha}{\beta}$ is a root to the polynomial $at^2 + bt + c = 0$.

Also, $\frac{\alpha}{\beta}$ is of the form $c_0 + c_1\sqrt[3]{2} + c_2\sqrt[3]{4}$ for some

rational numbers c_0, c_1, c_2 . Because it is a root to a quadratic with rational coefficients, applying the quadratic formula

shows that it must also be of the form $d + e\sqrt{f}$ for rationals d, e, f .

Thus, $(c_0 - d) + c_1\sqrt[3]{2} + c_2\sqrt[3]{4} = e\sqrt{f}$.

Writing $c'_0 = c_0 - d$, the quantity

$(c'_0 + c_1\sqrt[3]{2} + c_2\sqrt[3]{4})^2$ must be an integer. After

expanding this square, we obtain an expression of the form $\alpha + \beta\sqrt[3]{2} + \gamma\sqrt[3]{4}$ for some integers α, β, γ . This

expression is a quadratic in $\sqrt[3]{2}$ with integer coefficients. Because $x^3 - 2$ is irreducible over $\mathbb{Z}[x]$, it follows that

$\alpha = \beta = \gamma = 0$.

Thus, $0 = \beta = 2(c_2^2 + c'_0c_1)$ and $0 = \gamma = 2c'_0c_2 + c_1^2$.

From the first of these equations, $(c'_0c_1)^2 = -2c_2^3c_1$;

from the second, $(c'_0c_1)^2 = c_2^4$. Hence $-2c_2^3c_1 = c_2^4$.

This implies that either $c_2 = 0$ or $c_2 = -\sqrt[3]{2}c'_0$. In the latter case, because c_2 is rational we still must have $c_2 = c'_0 = 0$.

Then $c_1 = 0$ as well, and $\frac{\alpha}{\beta} = c_0$ is rational. Thus,

$(x, y) = \left(\frac{\alpha}{\beta}, 1\right)$ is a nonzero rational solution to the given equation.

3. (a) Yes: simply rotate the plane 90° about some axis perpendicular to it. For example, in the xy -plane we could map each point (x, y) to the point $(y, -x)$.

(b) Suppose such a bijection existed. Label the three-dimensional space with x -, y -, and z -axes. Given any point $P = (x_0, y_0, z_0)$, we also view it as the vector p from $(0, 0, 0)$ to (x_0, y_0, z_0) . The given condition says that $(a - b) \cdot (f(a) - f(b)) = 0$

for any vectors a, b .

Assume without loss of generality that f maps the origin to itself otherwise, $g(p) = f(p) - f(0)$ is still a bijection and still satisfies the above equation. Plugging $b = (0, 0, 0)$ into the equation above, we have $a \cdot f(a) = 0$ for all a . Then the above equation reduces to $a \cdot f(b) + b \cdot f(a) = 0$.

Given any vectors a, b, c and any reals m, n , we then have

$$m(a \cdot f(b) + b \cdot f(a)) = 0$$

$$n(a \cdot f(c) + c \cdot f(a)) = 0$$

$$a \cdot f(mb + nc) + (mb + nc) \cdot f(a) = 0.$$

Adding the first two equations and subtracting the third gives

$$a \cdot (mf(b) + nf(c) - f(mb + nc)) = 0.$$

Because this must be true for any vector a , we must have

$$f(mb + nc) = mf(b) + nf(c).$$

Therefore f is linear, and it is determined by how it transforms the unit vectors $i = (1, 0, 0)$, $j = (0, 1, 0)$, and $k = (0, 0, 1)$. If $f(i) = (a_1, a_2, a_3)$, $f(j) = (b_1, b_2, b_3)$, and $f(k) = (c_1, c_2, c_3)$, then for a vector x we have

$$f(x) = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} x.$$

Applying $f(a)$. $a = 0$ with $a = i, j, k$, we have

$$a_1 = b_2 = c_3 = 0.$$

Then applying $a \cdot f(b) + b \cdot f(a)$ with $(a, b) = (i, j)$, (j, k) , (k, i) we have $b_1 = -a_2$, $c_2 = -b_3$, $c_1 = -a_3$.

Setting $k_1 = c_2$, $k_2 = -c_1$, and $k_3 = b_1$ we find that $f(k_1 i + k_2 j + k_3 k) = k_1 f(i) + k_2 f(j) + k_3 f(k) = 0$.

Because f is injective and $f(0) = 0$, this implies that $k_1 = k_2 = k_3 = 0$.

Then $f(x) = 0$ for all x , contradicting the assumption that f was surjective. Therefore our original assumption was false, and no such bijection exists.

4. All angles are directed modulo 180° .

For convenience, call any triangle $A'B'C'$ proper if A', B', C' lie on lines AB, BC, CA , respectively, and $\triangle ABC \sim \triangle A'B'C'$. The problem is, then, to construct the proper triangle with minimal area.

Suppose we have any proper triangle, and let P be the point (different from A') where the circumcircles of triangles $AA'C'$ and $BB'A'$ meet.

$$\begin{aligned} \text{Then } \angle B'PC' &= 360^\circ - \angle A'PB' - \angle C'PA' \\ &= 360^\circ - (180^\circ - \angle CBA) - (180^\circ - \angle BAC) \\ &= 180^\circ - \angle ACB, \end{aligned}$$

so P also lies on the circumcircle of triangle $CC'B'$.

$$\text{Next, } \angle PAB = \angle PCA' = \angle B'CA' - \angle B'C'P$$

$$= \angle B'CC' - \angle B'C'P = \angle PCA,$$

and with similar reasoning we have

$$\angle PAB = \angle PCA' = \angle PCA = \angle PB'C' = \angle PBC.$$

There is a unique point P (one of the Brocard points) satisfying $\angle PAB = \angle PBC = \angle PCA$, and thus P is fixed – independent of the choice of triangle $A'B'C'$. Because P is the corresponding point in similar triangles ABC

$$\text{and } A'B'C', \text{ we have } [A'B'C'] = [ABC] \left(\frac{PA'}{PA} \right)^2$$

Thus $[A'B'C']$ is minimal when PA' is minimal, which occurs when $PA' \perp AB$ (and analogously, when $PB' \perp PC$ and $PC' \perp PA$). Thus, the proper triangle with minimal area is the pedal triangle $A'B'C'$ of P to triangle ABC . This triangle is indeed similar to triangle ABC : letting $\theta = \angle PAB$ be the Brocard angle, it is the image of triangle ABC under a rotation through

$\theta - 90^\circ$, followed by a homothety of ratio $|\sin \theta|$.

To construct this triangle, first draw the circles $\{x\} \angle BXA = \angle BCA + \angle CAB$ and $\{Y\} \angle CYB = \angle CAB + \angle ABC$ and let P' be their point of intersection (different from B). We have $\angle AP'C = \angle ABC + \angle BCA$, and $\angle PAB = 180^\circ - \angle ABP' - \angle BP'A = 180^\circ - (\angle ABC - \angle PBC) - (\angle BCA + \angle CAB) = \angle PBC$, and similarly $\angle PBC = \angle P'CA$. Therefore $P = P'$. Finally, drop the perpendiculars from P' to the sides of triangle ABC to form A', B', C' . This completes the construction.

5. We rewrite the equation in the form

$$(x - y)(x^2 + xy + y^2) = z^2$$

Any common divisor of $x - y$ and $x^2 + xy + y^2$ also divides both z^2 and $(x^2 + xy + y^2) - (x + 2y)(x - y) = 3y^2$. Because z^2 and $3y^2$ are relatively prime by assumption, $x - y$ and $x^2 + xy + y^2$ must be relatively prime as well. Therefore, both $x - y$ and $x^2 + xy + y^2$ are perfect squares.

Writing $a = \sqrt{x - y}$, we have

$$x^2 + xy + y^2 = (a^2 + y)^2 + (a^2 + y)y + y^2$$

$$= a^4 + 3a^2y + 3y^2$$

$$\text{and } 4(x^2 + xy + y^2) = (2a^2 + 3y)^2 + 3y^2.$$

$$\text{Writing } m = 2\sqrt{x^2 + xy + y^2} \text{ and } n = 2a^2 + 3y,$$

$$\text{we have } m^2 = n^2 + 3y^2 \text{ or } (m - n)(m + n) = 3y^2,$$

$$\text{so } (m - n, m + n) = (1, 3y^2), (y, 3y), \text{ or } (3, y^2)$$

In the first case, $2n = 3y^2 - 1$ and $4a^2 = 2n - 6y = 3y^2 - 6y - 1$.

Hence, $a^2 \equiv 2 \pmod{3}$, which is impossible.

In the second case, $n = y < 2a^2 + 3y = n$, a contradiction.

In the third case, we have

$$4a^2 = 2n - 6y = y^2 - 6y - 3 < (y - 3)^2.$$

When $y \geq 10$ we have $y^2 - 6y - 3 > (y - 4)^2$. Hence, we must actually have $y = 2, 3, 5$, or 7 . In this case

$$\text{we have } a = \frac{\sqrt{y^2 - 6y - 3}}{2}, \text{ which is real only when}$$

$y = 7$, $a = 1$, $x = y + a^2 = 8$, and $z = 13$. This yields the unique solution $(x, y, z) = (8, 7, 13)$.

6. Let the incircle of triangle ABD be tangent to $\overline{AD}$ at T_1 and let the incircle of triangle ACD be tangent to $\overline{AD}$ at T_2 . Then

$$DT_1 = \frac{1}{2}(DA + DB - AB) \text{ and } DT_2 = \frac{1}{2}(DA + DC - AC)$$

First suppose a circle can be inscribed inside AB_1DC_1 . Let it be tangent to sides AB_1, B_1D, DC_1, C_1A at points E, F, G, H , respectively. Using equal tangents, we have

$$\begin{aligned} AB - BD &= (AH + HB) - (BF - DF) \\ &= (AH + BF) - (BF - DF) = AH + DF \end{aligned}$$

and similarly $AC - CD = AE + DG$. Also, $AH + DF = AE + DG$ by equal tangents, implying that $AB - BD = AC - CD$ and thus $DA + DB - AB = DA + DC - AC$. Therefore $DT_1 = DT_2$, $T_1 = T_2$, and the two given incircles are tangent to each other.

Next suppose the two incircles are tangent to each other. Then $DA + DB - AB = DA + DC - AC$. Let ω be the incircle of ABB_1 , and let D' be the point on $\overline{BB_1}$ (different from B_1) such that line CD' is tangent to ω . Suppose, by way of contradiction, that $D \neq D'$. From the result in the last paragraph, we know that the incircles of triangles ABD' and ACD' are tangent and hence $D'A + D'B - AB = D'A + D'C - AC$. Because $DB - AB = DC - AC$ and $D'B - AB = D'C - AC$, we must have $DB - D'B = DC = D'C$ by subtraction. Thus $DD' = |DB - D'B| = |DC - D'C|$. The triangle inequality then fails in triangle $DD'C$, a contradiction. This completes the proof.

7. A binary string is a finite string of digits, all either 0 or 1. Call such a string (perhaps starting with zeroes) valid if it does not contain three consecutive equal digits. Let a_n represent the number of valid n -digit strings; let s_n be the number of valid strings starting with two equal digits; and let d_n be the number of valid strings starting with two different digits. Observe that $a_n = s_n + d_n$ for all n .

An $(n + 2)$ -digit string starting with 00 is valid if and only if its last n digits form a valid string starting with 1. Similarly, an $(n + 2)$ -digit string starting with 11 is valid if and only if its last n digits form a valid string starting with 0. Thus, $S_{n+2} = a_n = s_n + d_n$.

An $(n + 2)$ -digit string starting with 01 is valid if and only if its last n digits form a valid string starting with 00, 01, or 10. Similarly, an $(n + 2)$ -digit string starting with 10 is valid if and only if its last n digits form a valid string starting with 11, 01, or 10. Thus, $d_{n+2} = s_n + 2d_n$. Solving these recursions gives

$$s_{n+4} = 3s_n + 2 - s_n \text{ and } d_{n+4} = 3d_n + 2 - d_n,$$

which when added together yield

$$a_{n+4} = 3a_n + 2 - a_n.$$

Thus we can calculate initial values of a_n and then use the recursion to find other values.

n	1	2	3	4	5	6	7	8	9	10
a_n	2	4	6	10	16	26	42	68	110	178

Now of the a_n valid n -digit strings, only half start with 1. Thus only half are binary representations of positive numbers, and exactly

$$\frac{1}{2}(a_1 + a_2 + \dots + a_{10}) = 231$$

numbers between 1 and 1023 have the desired property. Ignoring 1, 2, and 3, we find that the answer is 228.

8. Assume without loss of generality that

$$x = \max\{x, y, z\}.$$

If $x \geq y \geq z$, then

$$x^2y + y^2z + z^2x \leq x^2y + y^2z + z^2x + z(xy + (x - y)(y - z))$$

$$= (x + z)^2 y = 4 \left(\frac{1}{2} - \frac{1}{2} y \right) \left(\frac{1}{2} - \frac{1}{2} y \right) y \leq \frac{4}{27}, \text{ where the}$$

last inequality follows from the AM-GM inequality. Equality occurs if and only if $z = 0$ (from the first

$$\text{inequality) and } y = \frac{1}{3}, \text{ in which case } (x, y, z) = \left(\frac{2}{3}, \frac{1}{3}, 0 \right).$$

If $x \geq z \geq y$, then $x^2y + y^2z + z^2x = x^2z + z^2y + y^2x - (x - z)(z - y)(x - y)$

$$\leq x^2z + z^2y + y^2x \leq \frac{4}{27}$$

where the second inequality is true from the result we proved for $x \geq y \geq z$ (except with y and z reversed). Equality holds in the first inequality only when two of x, y, z are equal, and in the second inequality only when

$$(x, z, y) = \left(\frac{2}{3}, \frac{1}{3}, 0 \right). \text{ Because these conditions can't both}$$

be true, the inequality is actually strict in this case.

Therefore the inequality is indeed true, and equality holds

$$\text{when } (x, y, z) \text{ equals } \left(\frac{2}{3}, \frac{1}{3}, 0 \right), \left(\frac{1}{3}, 0, \frac{2}{3} \right), \text{ or } \left(0, \frac{2}{3}, \frac{1}{3} \right).$$

9. Let the sides of the triangle have lengths $a \leq b \leq c$; let the angles opposite them be A, B, C ;

let the semiperimeter be $s = \frac{1}{2}(a + b + c)$; and let the

inradius be r . Without loss of generality, assume

that the triangle have circumradius $R = \frac{1}{2}$ and that

$$a = \sin A, b = \sin B, c = \sin C.$$

The area of the triangle equals both

$$rs = \frac{1}{2} r (\sin A + \sin B + \sin C) \text{ and}$$

$$abc/uR = \frac{1}{2} \sin A \sin B \sin C. \text{ Thus}$$

$$r = \frac{\sin A \sin B \sin C}{\sin A + \sin B + \sin C} \text{ and } \frac{a}{r} = \frac{\sin A + \sin B + \sin C}{\sin B + \sin C}$$

Because $A = 180^\circ - B - C$, $\sin A = \sin(B + C) = \sin B \cos C + \cos B \sin C$ and we also have

$$\frac{a}{r} = \cot B + \csc B + \cot C + \csc C$$

Note that $f(x) = \cot x + \csc x$ is a decreasing function along the interval $0^\circ < x < 90^\circ$. Now there are two cases; $B \leq 60^\circ$, or $B > 60^\circ$.

If $B > 60^\circ$, because $C > B > 60^\circ$, the triangle with $A' = B' = C' = 60^\circ$ has a larger ratio a'/r' . Therefore we may assume that $B \leq 60^\circ$. We may further assume that $A = B$ - otherwise, the triangle with angles

$$A' = B' = \frac{1}{2}(A+B) \leq B \text{ and } C' = C$$

has a larger ratio a'/r' . Because $C < 90^\circ$ we have $45^\circ < A \leq 60^\circ$. Now,

$$\frac{a}{r} = \frac{\sin A + \sin B + \sin C}{\sin B \sin C} = \frac{2 \sin A + \sin(2A)}{\sin A \sin(2A)} = 2 \csc(2A) + \csc A.$$

Now $\csc x$ has second derivative $\csc x (\csc^2 x + \cot^2 x)$, which is strictly positive when $0^\circ < x < 180^\circ$. Thus, both $\csc x$ and $\csc(2x)$ are strictly convex along the interval $0^\circ < x < 90^\circ$. Therefore, $g(A) = 2 \csc(2A) + \csc A$, a convex function in A , is maximized in the interval $45^\circ \leq A \leq 60^\circ$ at one of the endpoints. Because $g(45^\circ) = 2 + \sqrt{2} < 2\sqrt{3} = g(60^\circ)$, it is maximized when $A = B = C = 60^\circ$.

Therefore the maximum ratio is $2\sqrt{3}$, attained with an equilateral triangle.

10. Let α, β, γ be the three roots. Without loss of generality, suppose that $0 \leq \alpha \leq \beta \leq \gamma$. We have $x - a = x + \alpha + \beta + \gamma \geq 0$ and $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$. If $0 \leq x \leq \alpha$, then (applying the AM-GM inequality to obtain the first inequality below)

$$\begin{aligned} -f(x) &= (\alpha - x)(\beta - x)(\gamma - x) \leq \frac{1}{27}(\alpha + \beta + \gamma - 3x)^3 \\ &\leq \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3, \end{aligned}$$

so that $f(x) \geq -\frac{1}{27}(x - a)^3$. Equality holds exactly when $\alpha - x = \beta - x = \gamma - x$ in the first inequality and $\alpha + \beta + \gamma - 3x = x + \alpha + \beta + \gamma$ in the second; that is, when $x = 0$ and $\alpha = \beta = \gamma$.

If $\beta \leq x \leq \gamma$, then (again applying the AM-GM inequality to obtain the first inequality below)

$$\begin{aligned} -f(x) &= (x - \alpha)(x - \beta)(\gamma - x) \leq \frac{1}{27}(x + \gamma - \alpha - \beta)^3 \\ &\leq \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3, \end{aligned} \text{ so that again}$$

$f(x) \geq -\frac{1}{27}(x - a)^3$. Equality holds exactly when $x - \alpha = x - \beta = \gamma - x$ in the first inequality and $x + \gamma - \alpha - \beta = x + \alpha + \beta + \gamma$; that is, when $\alpha = \beta = 0$ and $\gamma = 2x$.

Finally, when $\alpha < x < \beta$ or $x > \gamma$ then

$$f(x) > 0 \geq -\frac{1}{27}(x - a)^3$$

Thus, $\lambda = -\frac{1}{27}$ works. From the above reasoning we can find that λ must be at most $-\frac{1}{27}$ or else the inequality fails for the polynomial $f(x) = x^2(x - 1)$ at $x = \frac{1}{2}$. Equality occurs either when $\alpha = \beta = \gamma$ and $x = 0$; or when $\alpha = \beta = 0$, γ is any nonnegative real, and $x = \frac{\gamma}{2}$.

11. We know that

$$m_a^2 = \frac{1}{4}(2b^2 + 2c^2 - a^2), \quad m_b^2 = \frac{1}{4}(2a^2 + 2c^2 - b^2)$$

$$\text{so } m_a^2 - m_b^2 = \frac{3}{4}(b^2 - a^2)$$

If the desired condition $m_a - m_b = b - a \neq 0$ is to be satisfied, then necessarily $m_a + m_b = \frac{3}{4}(b + a)$. Find the system of equations $m_a - m_b = b - a$

$$m_a + m_b = \frac{3}{4}(b + a)$$

we find that we would then have

$$m_a = \frac{1}{8}(7b - a), m_b = \frac{1}{8}(7a - b),$$

$$\text{and } a + m_a = b + m_b = \frac{7}{8}(a + b). \text{ Thus } k = \frac{7}{8}$$

Now we examine for what $a \neq b$ there exists a triangle ABC with sides a, b and medians

$m_a = \frac{1}{8}(7b - a), m_b = \frac{1}{8}(7a - b)$. We can find all three side lengths of the triangle AB_1G , where G is the centroid of the triangle ABC and B_1 is the midpoint of the side AC :

$$AB_1 = \frac{b}{2}, AG = \frac{2}{3}m_a = \frac{2}{3} \cdot \frac{1}{8}(7b - a) = \frac{1}{12}(7b - a)$$

$$B_1G = \frac{1}{3}m_b = \frac{1}{3} \cdot \frac{1}{8}(7a - b) = \frac{1}{24}(7a - b)$$

Examining the triangle inequalities for these three

lengths, we get the condition $\frac{1}{3} < \frac{a}{b} < 3$, from which the value $\frac{a}{b} = 1$ has to be excluded by assumption.

This condition is also sufficient: once the triangle AB_1G has been constructed, it can always be completed to a triangle ABC with $b = AC, m_a = AA_1, m_b = BB_1$. Then from the equality $m_a^2 - m_b^2 = \frac{3}{4}(b^2 - a^2)$ we would also have $a = BC$.

12. Because $10^{2n} \cdot 30^{2n}$ is even, x and y must have the same parity. Then (x, y) is a valid solution if and only if $(u, v) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$ is a pair of positive integers that satisfies $u > v$ and $uv = 5^{2n} \cdot 30^{2n}$. Now $5^{2n} \cdot 30^{2n} = 2^{2n} \cdot 3^{2n} \cdot 5^{2n+2}$ has exactly $(2n+1)^2(2n+3)$ factors. Thus without the condition $u > v$ there are exactly $(2n+1)^2(2n+3)$ such pairs (u, v) . Exactly one pair has $u = v$, and by symmetry half of the remaining pairs have $u > v$. It follows that there are

$$\frac{1}{2}((2n+1)^2(2n+3)-1) = (n+1)(4n^2+6n+1) \text{ valid pairs.}$$

Now suppose that $(n+1)(4n^2+6n+1)$ were a square. Because $n+1$ and $4n^2+6n+1 = (4n+2)(n+1)-1$ are coprime, $4n^2+6n+1$ must be a square as well. However, $(2n+1)^2 < 4n^2+6n+1 < (2n+2)^2$, a contradiction.

13. Make the substitution $y = \sqrt{x+1}$, so $y \in (0, 1) \cup (1, \infty)$ and $x = y^2 - 1$. Then the inequality is equivalent to

$$\frac{(y^2-1)^2}{(y^2-y)^2} < \frac{(y^2-1)^2+3(y^2-1)+18}{y^4}$$

$$\Leftrightarrow \frac{(y+1)^2}{y^2} < \frac{y^4+y^2+16}{y^4}$$

$\Leftrightarrow (y+1)^2 y^2 < y^4 + y^2 + 16 \Leftrightarrow 2y^3 < 16$, so the condition is satisfied exactly when $y < 2$; i.e., exactly when $y \in (0, 1) \cup (1, 2)$ and $x \in (-1, 0) \cup (0, 3)$.

14. By Ceva's Theorem, the cevians AD , BE , CF in $\triangle ABC$ are concurrent if and only if (using directed line segments)

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

In this problem, $\frac{AF}{FB} = 1$, and $\frac{CE}{EA} = \frac{a}{c}$. Thus AD , BE ,

CF are concurrent if and only if $\frac{BD}{DC} = \frac{c}{a}$.

This in turn is true if and only if

$$BD = \frac{ac}{a+c} \text{ and } CD = \frac{a^2}{a+c}.$$

Because $AB^2 - BD^2 = BD^2 = AC^2 - CD^2$, these last conditions hold exactly when the following equations are true:

$$AB^2 - \left(\frac{ac}{a+c}\right)^2 = AC^2 - \left(\frac{a^2}{a+c}\right)^2$$

$$(a+c)^2 c^2 - a^2 c^2 = (a+c)^2 b^2 - a^4$$

$$a^4 - a^2 c^2 = (b^2 - c^2)(a+c)^2$$

$$a^2(a-c) = (b^2 - c^2)(a+c)$$

Therefore the three lines are concurrent if and only if the given equation holds, as desired.

Alternatively, applying the law of cosines gives

$$\begin{aligned} \frac{BD}{DC} &= \frac{c \cos B}{b \cos C} = \frac{c}{b} \cdot \frac{a^2 + c^2 - b^2}{2ac} \cdot \frac{2ab}{a^2 + b^2 - c^2} \\ &= \frac{a^2 + c^2 - b^2}{a^2 + b^2 - c^2} \end{aligned}$$

Again, this equals $\frac{c}{a}$ exactly when the given equation holds,

$$\textbf{15. Set } t = \frac{x-3}{x+1} \text{ so that } x = \frac{t-3}{t+1} \text{ and } \frac{x-3}{x+1} = \frac{3+t}{1-t}.$$

Again we can rewrite the given equation, this time as

$$f\left(\frac{3+t}{1-t}\right) + f(t) = \frac{t-3}{t+1}$$

Adding these two equations we have

$$\frac{8t}{1-t^2} = 2f(t) + f\left(\frac{t-3}{t+1}\right) + f\left(\frac{3+t}{1-t}\right) = 2f(t) + t, \text{ so that}$$

$$f(t) = \frac{4t}{1-t^2} - \frac{t}{2}.$$

Adding these two equations we have

$$\frac{8t}{1-t^2} = 2f(t) + f\left(\frac{t-3}{t+1}\right) + f\left(\frac{3+t}{1-t}\right) = 2f(t) + t, \text{ so that}$$

$f(t) = \frac{4t}{1-t^2} - \frac{t}{2}$, and some algebra verifies that this solution works.

16. (a) No matter what x is, there is a unique value $\lambda = \frac{b-x}{b-a}$ such that $x = \lambda a + (1-\lambda)b$. Also,

$$0 < \frac{b-x}{b-a} < 1 \Leftrightarrow a < x < b, \text{ which proves the claim.}$$

(b) The condition given says that f is strictly convex. Stated geometrically, whenever $x < t < y$ the point $(t, f(t))$ lies strictly below the line joining $(x, f(x))$ and $(y, f(y))$. Suppose there were a parallelogram on the graph of f whose vertices, from left to right, have x -coordinates a, b, d, c . Then either $(b, f(d))$ or $(d, f(d))$ must lie on or above the line joining $(a, f(a))$ and $(c, f(c))$, a contradiction.



CBSE DRILL



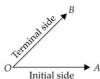
Synopsis and Chapter wise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Trigonometric Functions | Principle of Mathematical Induction

Trigonometric Functions

ANGLES

- The measure of rotation of a given ray about its initial point is called angle.
- Measure of an Angle:** It is the amount of rotation from the initial side to the terminal side.



DEGREE MEASURE

If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{\text{th}}$ of a revolution, the angle is said to have

a measure of one degree (1°).

$1^\circ = 60'$ (60 minutes),

$1' = 60''$ (60 seconds).

MEASUREMENT OF ANGLES

RADIAN MEASURE

The angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle is called a radian and is denoted by 1^c .

- Relation between Degree and Radian:**
- $1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ 16'$ (approximately)
- $1^\circ = \frac{\pi}{180} \text{ radian} = 0.01746 \text{ radian}$ (approximately)

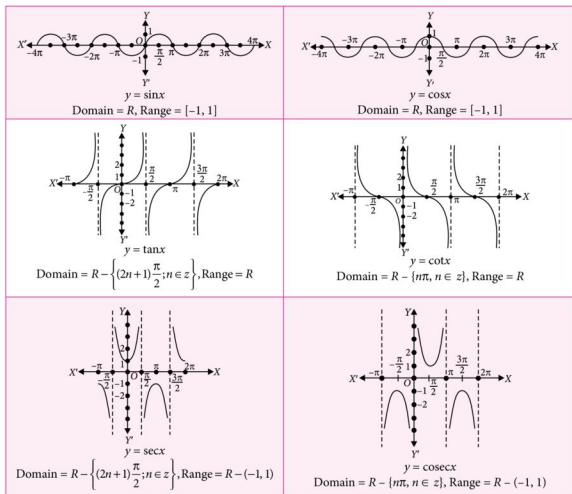
Note: If in a circle of radius r , an arc of length l , subtends

an angle θ radians at the centre, then $\theta = \frac{l}{r}$

SIGNS OF TRIGONOMETRIC FUNCTIONS:

	I	II	III	IV
$\sin x$	+	+	-	-
$\cos x$	+	-	-	+
$\tan x$	+	-	+	-
$\operatorname{cosec} x$	+	+	-	-
$\sec x$	+	-	-	+
$\cot x$	+	-	+	-

GRAPHS OF TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS OF SOME ALLIED ANGLES IN TERMS OF θ

T-Ratios Allied Angles	$\sin$	$\cos$	$\tan$	$\cot$	$\sec$	$\csc$
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$	$\sec\theta$	$-\csc\theta$
$\left(\frac{\pi}{2} \pm \theta\right)$	$\cos\theta$	$\mp \sin\theta$	$\mp \cot\theta$	$\mp \tan\theta$	$\mp \csc\theta$	$\sec\theta$
$(\pi \pm \theta)$	$\mp \sin\theta$	$-\cos\theta$	$\pm \tan\theta$	$\pm \cot\theta$	$-\sec\theta$	$\mp \csc\theta$
$\left(\frac{3\pi}{2} \pm \theta\right)$	$-\cos\theta$	$\pm \sin\theta$	$\mp \cot\theta$	$\mp \tan\theta$	$\pm \csc\theta$	$-\sec\theta$
$(2\pi \pm \theta)$	$\pm \sin\theta$	$\cos\theta$	$\pm \tan\theta$	$\pm \cot\theta$	$\sec\theta$	$\pm \csc\theta$

SOME IMPORTANT FORMULAE

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
- $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$
- $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$
 $= 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$
- $\cos 3A = 4 \cos^3 A - 3 \cos A$
- $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
- $\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$
- $\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$
- $\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$
- $\cos A - \cos B = 2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{B-A}{2} \right)$
- $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$
- $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$
- $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$
- $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$
- $\sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
- $\cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
- $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$
 - +ve, if $\frac{A}{2}$ lies in I or II quadrants
 - ve, if $\frac{A}{2}$ lies in III or IV quadrants

- $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$
 - +ve, if $\frac{A}{2}$ lies in I or IV quadrants
 - ve, if $\frac{A}{2}$ lies in II or III quadrants
- $\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$
 - +ve, if $\frac{A}{2}$ lies in I or III quadrants
 - ve, if $\frac{A}{2}$ lies in II or IV quadrants

TRIGONOMETRIC EQUATIONS AND ITS SOLUTIONS

Equations involving one or more than one trigonometric ratios of unknown angles are called trigonometric equations. The value of unknown angle that satisfies the given trigonometric equation is called solution or root of the equation.

Types of solutions

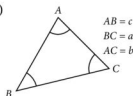
- Principal solution of a trigonometric equation :**
The solutions of a trigonometric equation lying in the interval $[0, 2\pi)$ are called principal solutions.
- General solutions of some trigonometric equations:**
The solution consisting of all possible solutions of a trigonometric equation is called its general solution.
- $\sin \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- $\cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
- $\tan \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$
- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$
- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in \mathbb{Z}$

PROPERTIES OF TRIANGLES

Sine Formula (sine rule)

The sines of the angles are proportional to the lengths of the opposite sides.

$$\text{i.e., } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Cosine Formula

- $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
- $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

• **Napoleon or Napier's Analogy or Tangent Law**

$$\begin{aligned} \bullet \quad \tan\left(\frac{A-B}{2}\right) &= \frac{a-b}{a+b} \cot \frac{C}{2} \\ \bullet \quad \tan\left(\frac{B-C}{2}\right) &= \frac{b-c}{b+c} \cot \frac{A}{2} \\ \bullet \quad \tan\left(\frac{C-A}{2}\right) &= \frac{c-a}{c+a} \cot \frac{B}{2} \end{aligned}$$

PRINCIPLE OF MATHEMATICAL INDUCTION

Mathematical statements :

Statements involving mathematical relations are known as mathematical statements.

It is denoted as $P(n)$.

First principle of mathematical induction :

Let $P(n)$ be the given statement involving the natural number n such that

(I) $P(1)$ is true i.e., $P(n)$ is true for $n = 1$ and

(II) $P(m+1)$ is true, whenever $P(m)$ is true i.e., $P(m)$ is true $\Rightarrow P(m+1)$ is true.

Then, $P(n)$ is true for all natural numbers n .

Second principle of mathematical induction :

Let $P(n)$ be the given statement involving the natural number n such that

(I) $P(1)$ is true i.e., $P(n)$ is true for $n = 1$ and

(II) $P(m+1)$ is true, whenever $P(n)$ is true for all n , where $1 \leq n \leq m$.

Then, $P(n)$ is true for all natural numbers.

WORK IT OUT

VERY SHORT ANSWER TYPE

- In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.
- If α and β are acute angles such that $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, then prove that $\alpha + \beta = \frac{\pi}{4}$.
- If $\cos \theta = -\frac{1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of $4 \tan^2 \theta - 3 \operatorname{cosec}^2 \theta$.
- Find the principal solutions of $\sin x = \frac{-\sqrt{3}}{2}$.
- Solve : $7 \cos^2 \theta + 3 \sin^2 \theta = 4$

SHORT ANSWER TYPE

- Prove that:
 $2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$
- Find the values of $\cos \theta$ and $\tan \theta$, if
 $\sin \theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$.

8. Prove that:

$$\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$$

9. If $A + B + C = \pi$, prove that

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$$

10. In any $\triangle ABC$, prove that

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$$

LONG ANSWER TYPE - I

- If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$, prove that $\tan 2\alpha = \frac{56}{33}$
- Solve : $4 \sin x \sin 2x \sin 4x = \sin 3x$
- Prove that : $1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$ for all $n \in \mathbb{N}$.
- Prove that : $\sin^2 A = \cos^2(A - B) + \cos^2 B - 2 \cos(A - B) \cos A \cos B$.
- Solve: $\sec x - \tan x = \sqrt{3}$.

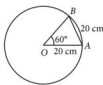
LONG ANSWER TYPE - II

- If $0 \leq x \leq 2\pi$, find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$, when:
 - $\tan x = -\frac{4}{3}$, x lies in quadrant II
 - $\cos x = -\frac{1}{3}$, x lies in quadrant III
 - $\sin x = -\frac{1}{2}$, x lies in quadrant IV
- Using the principle of mathematical induction, prove that
 $\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)} \forall n \in \mathbb{N}$
- Prove that:
 $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$
- Prove that: $2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24, for all $n \in \mathbb{N}$.
- Prove that:
 - $1 + \cos 2x + \cos 4x + \cos 6x = 4 \cos x \cos 2x \cos 3x$.
 - $(\sin 3A + \sin A) \sin A + (\cos 3A - \cos A) \cos A = 0$.

SOLUTIONS

- Let arc $AB = s$. It is given that $OA = 20$ cm and chord $AB = 20$ cm. Therefore, $\triangle OAB$ is an equilateral triangle. Hence,

$$\angle AOB = 60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$



$$\therefore \theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{\pi}{3} = \frac{s}{20} \Rightarrow s = \frac{20\pi}{3} \text{ cm.}$$

2. We have, $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \times \frac{1}{2m+1}}$$

$$= \frac{2m^2 + m + m + 1}{2m^2 + 3m + 1 - m} = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{4}$$

3. Since θ lies in the third quadrant. Therefore, $\sin \theta$ is negative and $\tan \theta$ is positive.

$$\text{Now, } \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

$$\Rightarrow \sin \theta = -\sqrt{1 - \frac{1}{4}} = -\frac{\sqrt{3}}{2} \Rightarrow \csc \theta = \frac{-2}{\sqrt{3}}$$

$$\text{And, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \sqrt{3}.$$

$$\text{Hence, } 4 \tan^2 \theta - 3 \csc^2 \theta = 4 \times 3 - 3 \times \frac{4}{3} = 8.$$

4. The given equation is $\sin x = \frac{-\sqrt{3}}{2}$.

$$\text{We know that } \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

$$\therefore \sin \left(\pi + \frac{\pi}{3} \right) = -\sin \frac{\pi}{3} = \frac{-\sqrt{3}}{2}, \text{ and}$$

$$\sin \left(2\pi - \frac{\pi}{3} \right) = -\sin \frac{\pi}{3} = \frac{-\sqrt{3}}{2}.$$

$$\therefore \sin \frac{4\pi}{3} = \frac{-\sqrt{3}}{2} \text{ and } \sin \frac{5\pi}{3} = \frac{-\sqrt{3}}{2}.$$

Hence, the principal solutions are

$$x = \frac{4\pi}{3} \text{ and } x = \frac{5\pi}{3}.$$

5. We have, $7 \cos^2 \theta + 3 \sin^2 \theta = 4$

$$\Rightarrow 7(1 - \sin^2 \theta) + 3 \sin^2 \theta = 4 \Rightarrow 4 \sin^2 \theta = 3$$

$$\Rightarrow \sin^2 \theta = \frac{3}{4} = \left(\frac{\sqrt{3}}{2} \right)^2$$

$$\Rightarrow \sin^2 \theta = \sin^2 \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

6. L.H.S. $= 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3}$

$$= 2 \left(\sin \frac{3\pi}{4} \right)^2 + 2 \left(\cos \frac{\pi}{4} \right)^2 + 2 \left(\sec \frac{\pi}{3} \right)^2$$

$$= 2 \left(\sin \frac{\pi}{4} \right)^2 + 2 \left(\cos \frac{\pi}{4} \right)^2 + 2 \left(\sec \frac{\pi}{3} \right)^2$$

$$\left[\because \sin \frac{3\pi}{4} = \sin \left(\pi - \frac{\pi}{4} \right) = \sin \frac{\pi}{4} \right]$$

$$= 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 2(2)^2 = 1 + 1 + 8 = 10 = \text{R.H.S}$$

7. We have, $\cos^2 \theta + \sin^2 \theta = 1 \Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$
In the third quadrant $\cos \theta$ is negative and $\tan \theta$ is positive.

$$\therefore \cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{3}{5} \times -\frac{5}{4} = \frac{3}{4}$$

8. Clearly, $3A = 2A + A$

$$\Rightarrow \tan 3A = \tan (2A + A)$$

$$\Rightarrow \tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$\Rightarrow \tan 3A (1 - \tan 2A \tan A) = \tan 2A + \tan A$$

$$\Rightarrow \tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A$$

$$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$$

9. We have, L.H.S. $= \cos 2A + \cos 2B + \cos 2C$

$$= 2 \cos (A + B) \cos (A - B) + 2 \cos^2 C - 1$$

$$= 2 \cos (\pi - C) \cos (A - B) + 2 \cos^2 C - 1$$

$$= -2 \cos C \cos (A - B) + 2 \cos^2 C - 1$$

$$= -2 \cos C [\cos (A - B) - \cos C] - 1$$

$$= -1 - 2 \cos C [\cos (A - B) - \cos (\pi - (A + B))]$$

$$= -1 - 2 \cos C [\cos (A - B) + \cos (A + B)]$$

$$= -1 - 2 \cos C [2 \cos A \cos B]$$

$$= -1 - 4 \cos A \cos B \cos C = \text{R.H.S.}$$

10. By the sine rule, we have:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k (\text{say})$$

$$\Rightarrow a = k \sin A, b = k \sin B \text{ and } c = k \sin C.$$

$$\therefore \text{L.H.S.} = a(\sin B - \sin C) + b(\sin C - \sin A)$$

$$+ c(\sin A - \sin B)$$

$$= k \sin A (\sin B - \sin C) + k \sin B (\sin C - \sin A)$$

$$+ k \sin C (\sin A - \sin B)$$

$$= k[(\sin A \sin B - \sin A \sin C) + (\sin B \sin C - \sin A \sin B)$$

$$+ (\sin A \sin C - \sin B \sin C)]$$

$$= k \times 0 = 0 = \text{R.H.S.}$$

11. It is given that α, β lie between 0 and $\pi/4$. Therefore, $-\pi/4 < \alpha - \beta < \pi/4$ and $0 < \alpha + \beta < \pi/2$. So, $\cos(\alpha - \beta)$ and $\sin(\alpha + \beta)$ are positive.

$$\text{Now, } \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$$\text{and, } \cos(\alpha - \beta) = \sqrt{1 - \sin^2(\alpha - \beta)}$$

$$\Rightarrow \cos(\alpha - \beta) = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$$

$$\therefore \tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{3/5}{4/5} = \frac{3}{4} \text{ and,}$$

$$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{5}{12}$$

$$\text{Now, } \tan 2\alpha = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \frac{56}{33}$$

$$\begin{aligned} 12. \text{ We have, } & 4 \sin x \sin 2x \sin 4x = \sin 3x \\ \Rightarrow & 4 \sin x \sin(3x - x) \cdot \sin(3x + x) = \sin 3x \\ \Rightarrow & 4[\sin x(\sin^2 3x - \sin^2 x)] = 3 \sin x - 4 \sin^3 x \\ \Rightarrow & 4 \sin x \sin^2 3x - 4 \sin^3 x = 3 \sin x - 4 \sin^3 x \\ \Rightarrow & 4 \sin x \sin^2 3x = 3 \sin x \Rightarrow \sin x(4 \sin^2 3x - 3) = 0 \\ \Rightarrow & \sin x = 0 \text{ or, } 4 \sin^2 3x - 3 = 0 \end{aligned}$$

$$\Rightarrow \sin x = 0 \text{ or, } \sin^2 3x = \frac{3}{4}$$

$$\text{Now, } \sin x = 0 \Rightarrow x = m\pi, n \in \mathbb{Z}$$

$$\text{And, } \sin^2 3x = \frac{3}{4} \Rightarrow \sin^2 3x = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \sin^2 3x = \sin^2 \frac{\pi}{3} \Rightarrow 3x = m\pi \pm \frac{\pi}{3}, m \in \mathbb{Z}$$

$$\Rightarrow x = \frac{m\pi}{3} \pm \frac{\pi}{9}$$

$$\text{Hence, } x = m\pi \text{ or, } x = \frac{m\pi}{3} \pm \frac{\pi}{9}, \text{ where } m, n \in \mathbb{Z}$$

13. Let $P(n)$ be the statement given by

$$P(n): 1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$$

$$P(1): 1^2 > \frac{1^3}{3}$$

$$\therefore 1^2 = 1 > \frac{1}{3} = \frac{1^3}{3} \therefore P(1) \text{ is true.}$$

Let $P(n)$ be true for $n = m$. Then,

$$1^2 + 2^2 + 3^2 + \dots + m^2 > \frac{m^3}{3}$$

We shall now prove that $P(m + 1)$ is true.

$$\text{i.e., } 1^2 + 2^2 + 3^2 + \dots + m^2 + (m + 1)^2 > \frac{(m + 1)^3}{3}$$

Now, $P(m)$ is true

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + m^2 > \frac{m^3}{3}$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + m^2 + (m + 1)^2 > \frac{m^3}{3} + (m + 1)^2$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + m^2 + (m + 1)^2 > \frac{1}{3}(m^3 + 3m^2 + 6m + 3)$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + m^2 + (m + 1)^2 >$$

$$\frac{1}{3}\{m^3 + 3m^2 + 3m + 1 + (3m + 2)\}$$

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + m^2 + (m + 1)^2 >$$

$$\frac{1}{3}\{(m + 1)^3 + (3m + 2)\} > \frac{(m + 1)^3}{3}$$

$$\Rightarrow P(m + 1) \text{ is true.}$$

Thus, $P(m)$ is true $\Rightarrow P(m + 1)$ is true.

Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in \mathbb{N}$.

$$\begin{aligned} 14. \text{ R.H.S.} &= \cos^2(A - B) + \cos^2 B - 2 \cos(A - B) \cos A \cos B \\ &= \cos^2 B + \cos^2(A - B) - 2 \cos(A - B) \cos A \cos B \\ &= \cos^2 B + \cos(A - B) \{\cos(A - B) - 2 \cos A \cos B\} \\ &= \cos^2 B + \cos(A - B) \{\cos A \cos B + \sin A \sin B \\ &\quad - 2 \cos A \cos B\} \\ &= \cos^2 B + \cos(A - B) \{\sin A \sin B - \cos A \cos B\} \\ &= \cos^2 B - \cos(A - B) \{\cos A \cos B - \sin A \sin B\} \\ &= \cos^2 B - \cos(A - B) \cos(A + B) = \cos^2 B - (\cos^2 A - \sin^2 B) \\ &= \cos^2 B + \sin^2 B - \cos^2 A = 1 - \cos^2 A = \sin^2 A = \text{L.H.S.} \end{aligned}$$

$$15. \text{ We have, } \sec x - \tan x = \sqrt{3} \Rightarrow \frac{1}{\cos x} - \frac{\sin x}{\cos x} = \sqrt{3}$$

$$\Rightarrow (1 - \sin x) = \sqrt{3} \cos x \Rightarrow \sqrt{3} \cos x + \sin x = 1 \dots (i)$$

Dividing both sides of (i) by 2, we get

$$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\Rightarrow \cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos \frac{\pi}{3}$$

$$\Rightarrow \left(x - \frac{\pi}{6}\right) = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$[\because \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha]$$

$$\Rightarrow \left(x - \frac{\pi}{6}\right) = \left(2n\pi + \frac{\pi}{3}\right) \text{ or } \left(x - \frac{\pi}{6}\right) = \left(2n\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{3} + \frac{\pi}{6} \text{ or } x = 2n\pi - \frac{\pi}{3} + \frac{\pi}{6}, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \left(2n\pi + \frac{\pi}{2}\right) \text{ or } x = \left(2n\pi - \frac{\pi}{6}\right), \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \left(2n\pi - \frac{\pi}{6}\right), \text{ where } n \in \mathbb{Z}$$

$$\left[\because \sec x \text{ is not defined when } x = \left(2n\pi + \frac{\pi}{2}\right) \right].$$

Hence, the general solution is $x = \left(2n\pi - \frac{\pi}{6}\right)$, where $n \in \mathbb{Z}$.

16. (i) Refer to answer 65, page no. 64 (MTG CBSE Champion Mathematics Class 11).

(ii) It is given that x lies in the III quadrant.

$$\text{i.e., } \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\Rightarrow \frac{x}{2}$ lies in IInd quadrant

$$\Rightarrow \cos \frac{x}{2} < 0, \sin \frac{x}{2} > 0 \text{ and } \tan \frac{x}{2} < 0$$

$$\therefore \cos \frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}} \quad \left[\because \cos \frac{x}{2} < 0 \right]$$

$$\Rightarrow \cos \frac{x}{2} = -\sqrt{\frac{1 - 1/3}{2}} = -\frac{1}{\sqrt{3}} \quad \left[\because \cos x = -\frac{1}{3} \right]$$

$$\text{and } \sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} \quad \left[\because \sin \frac{x}{2} > 0 \right]$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{1 + 1/3}{2}} = \sqrt{\frac{2}{3}} \quad \left[\because \cos x = -\frac{1}{3} \right]$$

$$\text{Also, } \tan \frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \sqrt{\frac{2}{3}} \times -\sqrt{3} = -\sqrt{2}$$

(iii) It is given that x lies in IVth quadrant in which $\cos x$ is positive.

$$\therefore \sin x = -\frac{1}{2} \Rightarrow \cos x = \sqrt{1 - \sin^2 x}$$

$$= \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Now, x lies in IVth quadrant

$$\Rightarrow \frac{3\pi}{2} < x < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi$$

$\Rightarrow \frac{x}{2}$ lies in IInd quadrant

$$\Rightarrow \cos \frac{x}{2} < 0, \sin \frac{x}{2} > 0 \text{ and } \tan \frac{x}{2} < 0$$

$$\therefore \cos \frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}} \quad \left[\because \cos \frac{x}{2} < 0 \right]$$

$$\Rightarrow \cos \frac{x}{2} = -\sqrt{\frac{1 + \sqrt{3}/2}{2}} = -\frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\text{and } \sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} \quad \left[\because \sin \frac{x}{2} > 0 \right]$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{1 - \sqrt{3}/2}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\text{and, } \tan \frac{x}{2} = \frac{\sin(x/2)}{\cos(x/2)} = \frac{\sqrt{2 - \sqrt{3}}}{2} \times \frac{-2}{\sqrt{2 + \sqrt{3}}}$$

$$= -\frac{\sqrt{2 - \sqrt{3}}}{2 + \sqrt{3}}$$

17. Refer to answer 30, page no. 89 (MTG CBSE Champion Mathematics Class 11).

18. We observed that

$$\sin^4 \frac{7\pi}{8} = \sin^4 \left(\pi - \frac{\pi}{8} \right) = \sin^4 \frac{\pi}{8} \text{ and,}$$

$$\sin^4 \frac{5\pi}{8} = \sin^4 \left(\pi - \frac{3\pi}{8} \right) = \sin^4 \frac{3\pi}{8}$$

$$\therefore \text{L.H.S.} = \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$$

$$= \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{\pi}{8}$$



COMIC CAPSULE

Happy Face Math



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$$\begin{aligned}
&= 2 \left\{ \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} \right\} = 2 \left\{ \left(\sin^2 \frac{\pi}{8} \right)^2 + \left(\sin^2 \frac{3\pi}{8} \right)^2 \right\} \\
&= 2 \left\{ \left[\frac{1 - \cos \frac{\pi}{4}}{2} \right]^2 + \left[\frac{1 - \cos \frac{3\pi}{4}}{2} \right]^2 \right\} \left[\because \frac{1 - \cos 2\theta}{2} = \sin^2 \theta \right] \\
&= \frac{2}{4} \left\{ \left(1 - \cos \frac{\pi}{4} \right)^2 + \left(1 - \cos \frac{3\pi}{4} \right)^2 \right\} \\
&= \frac{1}{2} \left\{ \left(1 - \frac{1}{\sqrt{2}} \right)^2 + \left(1 + \frac{1}{\sqrt{2}} \right)^2 \right\} \\
&= \frac{1}{2} \left\{ \left(1 + \frac{1}{2} - \sqrt{2} \right) + \left(1 + \frac{1}{2} + \sqrt{2} \right) \right\} = \frac{3}{2} = \text{R.H.S.}
\end{aligned}$$

19. Let $P(n)$ be the statement given by

$P(n) : 2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24.

We have, $P(1) : 2 \times 7^1 + 3 \times 5^1 - 5$ is divisible by 24

$\therefore 2 \times 7^1 + 3 \times 5^1 - 5 = 14 + 15 - 5 = 24$, which is divisible by 24.

$\therefore P(1)$ is true.

Let $P(m)$ be true. Then,

$2 \times 7^m + 3 \times 5^m - 5$ is divisible by 24

$\Rightarrow 2 \times 7^m + 3 \times 5^m - 5 = 24\lambda$ for some $\lambda \in \mathbb{N}$

$\Rightarrow 3 \times 5^m = 24\lambda + 5 - 2 \times 7^m$

Now, $2 \times 7^{m+1} + 3 \times 5^{m+1} - 5$

$= 2 \times 7^{m+1} + (3 \times 5^m) - 5$

...(i)

$$\begin{aligned}
&= 2 \times 7^{m+1} + (24\lambda + 5 - 2 \times 7^m) - 5 \quad [\text{Using (i)}] \\
&= 2 \times 7^{m+1} + 120\lambda + 25 - 10 \times 7^m - 5 \\
&= (2 \times 7^{m+1} - 10 \times 7^m) + 120\lambda + 20 \\
&= (2 \times 7 \times 7^m - 10 \times 7^m) + 120\lambda + 24 - 4 \\
&= (14 - 10)7^m - 4 + 24(5\lambda + 1) = 4(7^m - 1) + 24(5\lambda + 1) \\
&= 4 \times 6\mu + 24(5\lambda + 1) \quad [\because 7^m - 1 \text{ is a multiple of } 6 \text{ for all } m \in \mathbb{N} : 7^m - 1 = 6\mu, \mu \in \mathbb{N}] \\
&= 24(\mu + 5\lambda + 1), \text{ which is divisible by } 24. \\
&\therefore P(m+1) \text{ is true.}
\end{aligned}$$

Thus, $P(m)$ is true $\Rightarrow P(m+1)$ is true.

Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in \mathbb{N}$.

20. (i) L.H.S. $= 1 + \cos 2x + \cos 4x + \cos 6x$

$= (\cos 0x + \cos 2x) + (\cos 4x + \cos 6x)$

$= 2\cos x \cos x + 2\cos 5x \cos x = 2\cos x (\cos x + \cos 5x)$

$= 2\cos x (2 \cos 3x \cos 2x) = 4 \cos x \cos 2x \cos 3x$

$= \text{R.H.S.}$

(ii) We have,

L.H.S. $= (\sin 3A + \sin A) \sin A + (\cos 3A - \cos A) \cos A$

$= \left\{ 2 \sin \left(\frac{3A+A}{2} \right) \cos \left(\frac{3A-A}{2} \right) \right\} \sin A$

$+ \left\{ -2 \sin \left(\frac{3A+A}{2} \right) \sin \left(\frac{3A-A}{2} \right) \right\} \cos A$

$= 2 \sin 2A \cos A \sin A - 2 \sin 2A \sin A \cos A = 0 = \text{R.H.S.}$



CBSE to have Two Mathematics Question Papers for Class 9th to Class 12th : One easy and the other tough

With an aim to give students an opportunity to avoid the tag of being weak in Mathematics, the Central Board of Secondary Education (CBSE) is planning to introduce two versions for the Mathematics papers for classes 9th, 10th, 11th and 12th.

The Central Board of Secondary Education is examining a proposal to introduce two versions of the mathematics question paper so that students who aren't keen to take up the subject in higher studies don't have to sit for the tougher edition of the test. For instance, students who want to take up engineering and sit for the JEE Main and Advanced will be able to study mathematics at the higher level. Those who aspire to study medicine and sit for NEET will have the choice of studying the subject at the easier level.

The subject expert committee is examining the proposal and once it is approved by the board internally, it will be sent to NCERT and the University Grants Commission (UGC) for approval. To ensure that universities accept the two different levels of

mathematics being offered at the high school level, the proposal will also need to be approved by the UGC. The board has received representations to that effect from several schools and principals across the country, people familiar with the development said on condition of anonymity.

An official, noted that students aspiring to study economics or medicine don't need to study higher level mathematics at the high school level, but at present have no option. "Sometimes, this leads to their overall performance coming down. At least this would give students a choice," this person said. Once the proposal is approved by the CBSE internally, it will be shared with NCERT which is the body responsible for framing school curriculum.

A number of school boards overseas follow this practice. For instance, the Cambridge International General Certificate of Secondary Education (IGCSE) has a core curriculum in each subject which is generally within the ability range of a large majority of the students. It also offers an extended curriculum that is designed for students who are more

academically able. International Baccalaureate (IB) also provides this flexibility. Explaining the proposal, a person familiar with the development said, the curriculum would be designed in such a manner that students opting for higher mathematics will have to study, say calculus, but those opting for the easier level will not have to.

The Human Resource Development (HRD) ministry also formed a panel earlier this year to suggest measures to combat a maths phobia among students after a national survey of state-run schools revealed a "fear complex" among students on the subject. The panel was formed by the HRD minister Prakash Javadekar, who had earlier said: "One thing has come up in the National Assessment Survey 2017 that students have fear complex about maths. We have formed a committee under Gujarat education minister Bhupendrasinh Chudasama to see how to remove that fear complex so that the students see it as a friendly subject."



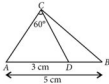
This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Trigonometric Functions

Total Marks : 80

Time Taken : 60 Min.

Only One Option Correct Type

- In a $\triangle ABC$ if $(\sqrt{3}-1)a = 2b$, $A = 3B$, then C is
(a) 60° (b) 120° (c) 30° (d) 45°
- The maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos\left(\frac{\pi}{4} - \theta\right)$ for real values of θ is
(a) 3 (b) 5
(c) 4 (d) none of these
- In the figure, ABC is a triangle in which $\angle C = 90^\circ$ and $AB = 5$ cm. D is a point on AB such that $AD = 3$ cm and $\angle ACD = 60^\circ$. Then the length of AC is

(a) $5\sqrt{\frac{3}{7}}$ cm (b) $\sqrt{\frac{7}{3}}$ cm
(c) $\frac{\sqrt{3}}{7}$ cm (d) none of these
- If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then the general value of θ is
(a) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$
(c) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (d) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$
- If $\tan \theta \cdot \tan(120^\circ - \theta) \cdot \tan(120^\circ + \theta) = \frac{1}{\sqrt{3}}$, then $\theta =$
(a) $\frac{n\pi}{3} + \frac{\pi}{18}, n \in \mathbb{Z}$ (b) $\frac{n\pi}{3} + \frac{\pi}{12}, n \in \mathbb{Z}$
(c) $\frac{n\pi}{12} + \frac{\pi}{12}, n \in \mathbb{Z}$ (d) $\frac{n\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z}$
- The value of $\cot 16^\circ \times \cot 44^\circ + \cot 44^\circ \times \cot 76^\circ - \cot 76^\circ \times \cot 16^\circ$ is
(a) 4 (b) 1 (c) 3 (d) 0

One or More Than One Option(s) Correct Type

- If $\sin A = a \cos B$ and $\cos A = b \sin B$, then $(a^2 - 1) \tan^2 A + (1 - b^2) \tan^2 B$ is equal to
(a) $\frac{a^2 - b^2}{b^2}$ (b) $\frac{a^2 - b^2}{a^2}$
(c) $\frac{a^2 + b^2}{b^2}$ (d) $\frac{a^2 + b^2}{a^2}$
- If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$, then which of the following is/are true?
(a) $\cos(A - B) = \frac{1}{3}$
(b) $|\cos A - \cos B| = \sqrt{\frac{2}{3}}$
(c) $\cos(A - B) = -\frac{1}{3}$
(d) $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

Solution Sender of Maths Musing

SET-185

- Sannibha Pande (West Bengal)
- Santanu Singh (West Bengal)

SET-186

- N. Jayanthi (Hyderabad)
- Kilaparathi Thrinadha (Andhra Pradesh)
- Devjit Acharee (West Bengal)

9. If $0 < \alpha, \beta < \pi$ and $\cos \alpha + \cos \beta - \cos(\alpha + \beta) = 3/2$, then

- (a) $\alpha = \pi/3$ (b) $\beta = \pi/3$
(c) $\alpha \neq \beta$ (d) $\alpha + \beta = \pi/3$

10. If $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = 1/2$ where $\alpha, \beta \in [0, \pi/2]$ then

- (a) $\tan(\alpha + 2\beta) = -\sqrt{3}$
(b) $\tan(2\alpha + \beta) = -1/\sqrt{3}$
(c) $\tan(\alpha + 2\beta) = \sqrt{3}$
(d) $\tan(2\alpha + \beta) = 1/\sqrt{3}$

11. If $\cos^3 x \sin 2x = \sum_{m=1}^n a_m \sin mx$ is an identity in x , then which one of the following is correct?

- (a) $a_2 = 0, a_3 = \frac{3}{8}$ (b) $a_1 = \frac{1}{2}, n = 6$
(c) $a_1 = \frac{1}{4}, n = 5$ (d) $\sum_{m=1}^n a_m = \frac{3}{4}$

12. If sides of triangle ABC are a, b and c such that $2b = a + c$, then

- (a) $\frac{b}{c} > \frac{2}{3}$ (b) $\frac{b}{c} > \frac{1}{3}$
(c) $\frac{b}{c} < 2$ (d) $\frac{b}{c} < \frac{1}{2}$

13. If $4\sin^4 x + \cos^4 x = 1$, then x is equal to

- (a) $n\pi, n \in \mathbb{Z}$ (b) $n\pi \pm \sin^{-1} \sqrt{\frac{2}{5}}, n \in \mathbb{Z}$
(c) $\frac{2n\pi}{3}, n \in \mathbb{Z}$ (d) $2n\pi \pm \frac{\pi}{4}, n \in \mathbb{Z}$

Comprehension Type

Given that $\cos \frac{\pi}{7}, \cos \frac{3\pi}{7}, \cos \frac{5\pi}{7}$ are the roots of the equation $8x^3 - 4x^2 - 4x + 1 = 0$.

14. The value of $\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14}$ is

- (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{7}}{8}$

15. The value of $\cos \frac{\pi}{14} \cos \frac{3\pi}{14} \cos \frac{5\pi}{14}$ is

- (a) $\frac{1}{4}$ (b) $\frac{1}{8}$
(c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{7}}{8}$

Matrix Match Type

16. Match the following:

Column-I	Column-II
P. $\text{Max}[5 \sin \theta + 3 \sin(\theta - \alpha), \theta \in \mathbb{R}] = 7$, then the set of possible values of α is	1. $2n\pi + 3\pi/4, n \in \mathbb{Z}$
Q. If $x \neq \frac{(2n+1)\pi}{2}$ and $(\cos x)^{(\sin^2 x - 3 \sin x + 2)} = 1$, then $x =$	2. $2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$
R. If $\sqrt{\sin x} + 2^{1/4} \cos x = 0$, then $x =$	3. $2n\pi + \cos^{-1}(1/3), n \in \mathbb{Z}$
S. If $\log_5 \tan x = (\log_5 4)(\log_4(3 \sin x))$, then $x =$	4. $2n\pi, n \in \mathbb{Z}$

P	Q	R	S
(a) 1	3	4	2
(b) 2	4	1	3
(c) 4	2	3	1
(d) 4	1	2	3

Integer Answer Type

17. If $\cos 5\theta = a \cos^5 \theta + b \cos^3 \theta + c \cos \theta$, then $c =$
 18. If $3 \sin x + 4 \cos x = 5$, then the value of $90 \tan^2(x/2) - 60 \tan(x/2) + 16$ is equal to
 19. If $k \left[\tan^6 \frac{\pi}{9} - 33 \tan^4 \frac{\pi}{9} + 27 \tan^2 \frac{\pi}{9} \right] = 9$, then the value of k is
 20. If $A + B + C = \pi$ and $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = \lambda \sin \left(\frac{A}{2} \right) \sin \left(\frac{B}{2} \right) \sin \left(\frac{C}{2} \right)$, then the value of λ must be

Keys are provided in this issue. Search now! ☺

SELF CHECK

No. of questions attempted
 No. of questions correct
 Marks scored in percentage

Check your score! If your score is

- > 90% EXCELLENT WORK! You are well prepared to take the challenge of final exam.
 90-75% GOOD WORK! You can score good in the final exam.
 74-60% SATISFACTORY! You need to score more next time.
 < 60% NOT SATISFACTORY! Revise thoroughly and strengthen your concepts.

CBSE DRILL



Synopsis and Chapter wise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Matrices and Determinants

MATRICES

A matrix is a rectangular array of mn numbers or functions with m horizontal lines (rows) and n vertical lines (columns) enclosed by $[]$ or $()$ is called a matrix of order $m \times n$.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n} = [a_{ij}]_{m \times n}$$

is a matrix of order $m \times n$.

ELEMENTS

The numbers or functions occurring in any matrix are called elements of the matrix.

In general, we have,

EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are said to be equal if

- they have the same order, i.e., the number of rows and columns in both the matrices are same.
- their corresponding elements are equal, i.e., $a_{ij} = b_{ij}$ for each i and j .

TYPES OF MATRICES

1.	Row matrix	A matrix having exactly one row and any number of columns is called a row matrix.
2.	Column matrix	A matrix having exactly one column and any number of rows is called a column matrix.
3.	Square matrix	A matrix having same number of rows and columns is called a square matrix.
4.	Diagonal matrix	A square matrix is called a diagonal matrix if all its non-diagonal elements are zero.
5.	Scalar matrix	A square matrix is called a scalar matrix if its non-diagonal elements are zero and diagonal elements are equal.

6.	Unit matrix or Identity matrix	A square matrix is called a unit matrix if its non-diagonal elements are zero and diagonal elements are all equal to one (unity).
7.	Null (or zero) matrix	A matrix having all the elements zero is called a null matrix or zero matrix. The zero matrix of order $m \times n$ is denoted by $O_{m \times n}$.
8.	Upper triangular matrix	A square matrix is called an upper triangular matrix if all the elements below the principal diagonal are zero i.e., the square matrix $[a_{ij}]_{m \times n}$ is an upper triangular matrix if $a_{ij} = 0$ for all $i > j$.
9.	Lower triangular matrix	A square matrix is called a lower triangular matrix if all the elements above the principal diagonal are zero i.e., the square matrix $[a_{ij}]_{m \times n}$ is a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

ALGEBRA OF MATRICES AND ITS PROPERTIES

Operations	Definition	Properties
Addition of two matrices	Let A and B be two matrices each of order $m \times n$. Then, $A + B = [a_{ij} + b_{ij}]$ $\forall i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$	(i) Commutative law : For any two matrices A and B , $A + B = B + A$ (ii) Associative law : For any three matrices A , B and C , $A + (B + C) = (A + B) + C$ (iii) Existence of Additive Identity : Let A be any matrix and O be a zero matrix, then $A + O = A = O + A$ i.e., O is the additive identity. (iv) Existence of Additive Inverse : For any matrix A , there exists a matrix $(-A)$ such that $A + (-A) = O = (-A) + A$. i.e., $(-A)$ is the additive inverse of A .
Multiplication of a matrix by a scalar	Let A be a matrix of order $m \times n$. Then, for any scalar k , $kA = [k \cdot a_{ij}]_{m \times n}$	Let A and B be any two matrices and k and l are scalars, then (i) $k(A + B) = kA + kB$ (ii) $(k + l)A = kA + lA$
Multiplication of two matrices	Let A and B be any two matrices of orders $m \times n$ and $n \times p$ respectively. Then $AB = C = [c_{ik}]_{m \times p}$ where $c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \dots + a_{in}b_{nk}$	(i) Multiplication of two matrices is not commutative. (ii) Associative law : For any three matrices A , B and C , $(AB)C = A(BC)$ (iii) Distributive law : For any three matrices A , B and C , $A(B + C) = AB + AC$, $(A + B)C = AC + BC$ (iv) Existence of multiplicative identity : For any square matrix, there exists a matrix I such that $AI = A = IA$ where I is called the identity matrix.

Note :

If A and B are two matrices of order $m \times n$, then

(i) $A - B = A + (-B)$

(ii) $(-A) = [-a_{ij}]_{m \times n}$, where $A = [a_{ij}]_{m \times n}$.

TRANPOSE OF A MATRIX

For any matrix $A = [a_{ij}]_{m \times n}$, the transpose is obtained by interchanging its rows and columns. It is denoted by A' or A^T i.e., $A' = [a_{ji}]_{n \times m}$

PROPERTIES OF TRANSPOSE

- $(A')' = A$
- $(kA)' = kA'$, where k is any constant
- $(A + B)' = A' + B'$
- $(AB)' = B'A'$

SOME SPECIAL MATRICES

- **Symmetric matrix** : A square matrix $A = [a_{ij}]$ is called a symmetric matrix, if $A = A'$ i.e., $a_{ij} = a_{ji} \forall i, j$.

- **Skew-symmetric matrix** : A square matrix $A = [a_{ij}]$ is called a skew-symmetric matrix if $A' = -A$ i.e., $a_{ij} = -a_{ji} \forall i, j$.

Note : (i) All the diagonal elements of a skew-symmetric matrix are zero.

(ii) For any square matrix A , $A + A'$ is symmetric matrix and $A - A'$ is skew-symmetric matrix.

(iii) Any square matrix can be expressed as the sum of symmetric and skew-symmetric matrix, i.e.

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

- **Orthogonal matrix** : A square matrix A is called an orthogonal matrix if $AA' = A'A = I$.

ELEMENTARY OPERATION (TRANSFORMATION) OF A MATRIX

The following three operations applied on the rows (columns) of a matrix are called elementary row (column) operations:

- Interchange of any two rows (columns)** : If i^{th} row (column) is interchanged with j^{th} row (column), we write $R_i \leftrightarrow R_j$ ($C_i \leftrightarrow C_j$).
- Multiplying the elements of a row (column) by a non-zero scalar** : If the elements of i^{th} row (column) are multiplied by a non-zero scalar k , we write $R_i \rightarrow kR_i$ ($C_i \rightarrow kC_i$).
- Adding to the elements of a row (column), the constant times the corresponding elements of another row (column)** : If k times the elements of j^{th} row (column) are added to the corresponding elements of the i^{th} row (column), we write $R_i \rightarrow R_i + kR_j$ ($C_i \rightarrow C_i + kC_j$).

INVERTIBLE MATRICES

A square matrix A of order n is said to be invertible if there exists a matrix B of same order such that, $AB = BA = I_n$. Here, B is called the inverse of A and is denoted by A^{-1} .

Note : (i) Inverse of any matrix, if exists, is unique.

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

INVERSE OF A MATRIX BY ELEMENTARY OPERATIONS

Let A be an invertible square matrix of order n . Then, $A = I_n A = IA$

Now we applying elementary row operation on matrix equation $A = IA$ till L.H.S. becomes I . Then the second

factor of R.H.S. i.e., A will remain same and first factor will be some matrix B such that $I = BA$. Hence, by definition, $B = A^{-1}$

DETERMINANTS

Corresponding to every square matrix A , there exists a number called the determinant of A and denoted by $|A|$.

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \text{ Then,}$$

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Note : Here, we have calculated determinant by expanding along first row.

Determinant can also be calculated by expanding matrix along any row or column.

PROPERTIES OF DETERMINANTS

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by -1 .
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

AREA OF A TRIANGLE

Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

MINORS AND COFACTORS

For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the row and the column of the element a_{ij} , then the determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij} . The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and denoted by A_{ij} .

$$\therefore A_{ij} = (-1)^{i+j} M_{ij}$$

ADJOINT OF A MATRIX

Adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as transpose of the matrix $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of the element a_{ij} .

PROPERTIES OF ADJOINT OF A MATRIX

- $A(\text{adj } A) = (\text{adj } A)A = |A|I_n$
- $\text{adj } (AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$, where n is the order of A .
- $\text{adj } (\text{adj } A) = |A|^{n-2}A \Rightarrow |\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$

INVERSE OF A MATRIX

For any square matrix A , inverse of A is defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

Note: (i) If $|A| = 0$, then the matrix is singular.

(ii) If $|A| \neq 0$, then the matrix is non-singular.

(iii) A square matrix is invertible if and only if A is non-singular matrix.

PROPERTIES OF INVERSE

- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})^T$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

SOLUTION OF SYSTEM OF LINEAR EQUATIONS

- If $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

then these equations can be written as $AX = B$, where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

- If $|A| \neq 0$, there exists unique solution given by $X = A^{-1}B$.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then there exists no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then system may or may not be consistent.

WORK IT OUT

VERY SHORT ANSWER TYPE

- Construct a 3×2 matrix whose elements are given by $a_{ij} = (i + 2j)$.
- Find x, y, z, w if $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.
- Find the value of k in order that the points $(5, 5)$, $(k, 1)$ and $(10, 7)$ are collinear.
- Find the value of x and y , given that $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

- Prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$.

SHORT ANSWER TYPE

- If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the determinant of the matrix $A^2 - 2A$.
- If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$.
- Determine the values of x for which the matrix $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$ is singular.
- If $A = \begin{bmatrix} 3 & 5 \\ 7 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -4 \\ 2 & 3 \end{bmatrix}$, find $(4A - 3B)$.
- If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$, find AB .

Does BA exist?

LONG ANSWER TYPE - I

- Let $A = \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ -2 & 2 \\ 1 & 6 \end{bmatrix}$.

Verify that $(A + B) + C = A + (B + C)$.

- Find A^{-1} , where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$.

- Show that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$$

MPP-3 CLASS XI ANSWER KEY

- (b)
- (c)
- (a)
- (a)
- (a)
- (c)
- (a)
- (b, c)
- (a, b)
- (a, c, d)
- (a, c)
- (a, b)
- (b)
- (d)
- (b)
- (5)
- (6)
- (3)
- (8)

14. Let $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix

of order 2.

Show that $(I + A) = (I - A) \cdot \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

15. By using elementary row transformations, find the

inverse of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

LONG ANSWER TYPE - II

16. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x and y such that $(xI + yA)^2 = A$.

17. Solve for x : $\begin{bmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{bmatrix} = 0$.

18. An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method.

19. Find the matrix A such that $|A| = 2$ and

$\text{adj } A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

20. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. Use this result to find A^5 .

SOLUTIONS

1. $a_{ij} = (i + 2j)$ for $i = 1, 2, 3$ and $j = 1, 2$
 $\therefore a_{11} = (1 + 2 \times 1) = 3$; $a_{12} = (1 + 2 \times 2) = 5$;
 $a_{21} = (2 + 2 \times 1) = 4$; $a_{22} = (2 + 2 \times 2) = 6$;
 $a_{31} = (3 + 2 \times 1) = 5$; $a_{32} = (3 + 2 \times 2) = 7$.

Hence, the required matrix $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 5 & 7 \end{bmatrix}$

2. Given $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

$\Rightarrow x - y = -1$, $2x - y = 0$, $2x + z = 5$ and $3z + w = 13$
 Solving the first two equations, we get $x = 1$ and $y = 2$
 Putting $x = 1$ in $2x + z = 5$, we get $z = 3$

Putting $z = 3$ in $3z + w = 13$, we get $w = 4$

$\therefore x = 1$, $y = 2$, $z = 3$ and $w = 4$

3. The given points are collinear

$\therefore \begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 5 & 5 & 1 \\ k-5 & -4 & 0 \\ 5 & 2 & 0 \end{vmatrix} = 0$

[Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$]

$\Rightarrow 1 \cdot [2(k-5) + 20] = 0$

$\Rightarrow 2k + 10 = 0 \Rightarrow k = -5$

4. We have, $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x+2y \\ 3y+2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\Rightarrow x + 2y = 3$... (i) $2x + 3y = 5$... (ii)

Solving (i) and (ii), we get $y = 1$, $x = 1$

5. Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$
 [Applying $C_3 \rightarrow C_3 + C_2$]

$= (a+b+c) \cdot \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ [Taking $(a+b+c)$ common from C_3]

$= (a+b+c) \times 0 = 0$ [$\because C_1$ and C_3 are identical].

6. $A^2 - 2A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+6 & 3+3 \\ 2+2 & 6+1 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$\therefore |A^2 - 2A| = \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix} = 25 - 0 = 25$

7. We have, $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow 2x + 1 = 5$ and $8 + y = 0 \Rightarrow x = 2$ and $y = -8$

$\therefore x - y = 2 - (-8) = 10$

8. For the matrix A to be singular $|A| = 0$

$\Rightarrow \begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$

$\Rightarrow (x+1) \{ (x+2)(x-6) - 2 \} + 3 \{ -5x + 30 - 8 \} +$

$4 \{ -5 - 4x - 8 \} = 0$

$\Rightarrow (x+1)(x^2 - 4x - 14) + 3(-5x + 22) + 4(-4x - 13) = 0$

$\Rightarrow x(x^2 - 3x - 49) = 0 \Rightarrow x = 0$, $x = \frac{1}{2}(3 \pm \sqrt{205})$

9. We have, $4A - 3B = 4A + (-3B)$

$$= \begin{bmatrix} 12 & 20 \\ 28 & -36 \end{bmatrix} + \begin{bmatrix} -18 & 12 \\ -6 & -9 \end{bmatrix} \\ = \begin{bmatrix} 12+(-18) & 20+12 \\ 28+(-6) & -36+(-9) \end{bmatrix} = \begin{bmatrix} -6 & 32 \\ 22 & -45 \end{bmatrix}$$

10. Here, A is a 3×2 matrix and B is a 2×2 matrix. Clearly, the number of columns in A equals the number of rows in B .

$\therefore AB$ exists and it is a 3×2 matrix.

$$\text{Now, } AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2-2 & 6-1 \\ -3+8 & 9+4 \\ -1+10 & 3+5 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & 13 \\ 9 & 8 \end{bmatrix}$$

Further, B is a 2×2 matrix and A is a 3×2 matrix. So, the number of columns in B is not equal to the number of rows in A . So, BA does not exist.

11. Clearly, each one of the matrices A, B, C is a (3×2) matrix. So, $(A+B)+C$ and $A+(B+C)$ are both defined and each one is a 3×2 matrix.

$$\text{Now, } (A+B) = \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & 6 \\ 0 & 5 \end{bmatrix}$$

$$\therefore (A+B)+C = \begin{bmatrix} 4 & -1 \\ 5 & 6 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & 2 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 3 & 8 \\ 1 & 11 \end{bmatrix}$$

$$\text{Also, } (B+C) = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & 2 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ -2 & 4 \\ -2 & 11 \end{bmatrix}$$

$$\therefore A+(B+C) = \begin{bmatrix} 1 & -2 \\ 5 & 4 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 4 \\ -2 & 4 \\ -2 & 11 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 3 & 8 \\ 1 & 11 \end{bmatrix}$$

Hence, $(A+B)+C = A+(B+C)$.

12. $|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix} = -6 + 28 + 45 = 67 \neq 0$

So, A is invertible.

$$\therefore \text{adj } A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

The given system of equations is

$$x + 2y - 3z = -4, 2x + 3y + 2z = 2, 3x - 3y - 4z = 11.$$

The given system of equation can be written as $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

Since A^{-1} exists, So system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\Rightarrow x = 3, y = -2$ and $z = 1$ is the required solution.

13. Refer to answer 55, page no. 72 (MTG CBSE Champion Mathematics Class 12).

14. Let $\tan \frac{\alpha}{2} = t$.

$$\text{Then, } \cos \alpha = \frac{1 - \tan^2(\alpha/2)}{1 + \tan^2(\alpha/2)} = \frac{1 - t^2}{1 + t^2}$$

$$\text{and } \sin \alpha = \frac{2 \tan(\alpha/2)}{1 + \tan^2(\alpha/2)} = \frac{2t}{1 + t^2}$$

$$\therefore (I+A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$\text{and } (I-A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$\text{Now, } (I-A) \cdot \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} & \frac{-2t}{1+t^2} + \frac{t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} & \frac{2t^2}{1+t^2} + \frac{1-t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I+A).$$

$$\text{Hence, } (I+A) = (I-A) \cdot \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

15. We have, $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\text{Applying } R_2 \rightarrow R_2 - 2R_1 \\ \Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_3, R_2 \rightarrow R_2 - 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

16. We have, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$\therefore xI + yA = x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + y \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow xI + yA = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix} = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Now, $(xI + yA)^2 = A$ (Given)

$$\Rightarrow \begin{bmatrix} x & y \\ -y & x \end{bmatrix} \begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 - y^2 & 2xy \\ -2xy & x^2 - y^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\Rightarrow x^2 - y^2 = 0, 2xy = 1, -2xy = -1$ and $x^2 - y^2 = 0$
 $\Rightarrow x^2 - y^2 = 0$ and $2xy = 1$. Now two cases arise.

Case I : When $x = y$ and $2xy = 1$

$$\Rightarrow 2x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$\therefore x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}} \text{ or } x = -\frac{1}{\sqrt{2}}, y = -\frac{1}{\sqrt{2}}$$

Case II : When $x = -y$ and $2xy = 1$

$$\Rightarrow -2x^2 = 1 \Rightarrow x = \pm \frac{i}{\sqrt{2}}$$

$$\therefore x = \frac{i}{\sqrt{2}}, y = -\frac{i}{\sqrt{2}} \text{ or } x = -\frac{i}{\sqrt{2}}, y = \frac{i}{\sqrt{2}}$$

17. Refer to answer 33, page no. 68 (MTG CBSE Champion Mathematics Class 12).

18. Let x, y and z ₹ be the investments at the rates of interest of 6%, 7% and 8% per annum respectively. Then, Total investment = ₹ 5000

$$\Rightarrow x + y + z = 5000$$

$$\text{Income from first investment of ₹ } x = ₹ \frac{6x}{100}$$

$$\text{Income from second investment of ₹ } y = ₹ \frac{7y}{100}$$

$$\text{Income from third investment of ₹ } z = ₹ \frac{8z}{100}$$

$$\therefore \text{ Total annual income} = ₹ \left(\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} \right)$$

$$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$$

$$[\because \text{ Total annual income} = ₹ 358]$$

$$\Rightarrow 6x + 7y + 8z = 35800$$

$$\text{Also, } \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow 6x + 7y - 8z = 7000$$

Thus, we obtain the following system of simultaneous linear equations :

$$x + y + z = 5000; 6x + 7y + 8z = 35800; 6x + 7y - 8z = 7000$$

The system of equations can be written as $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

$$= 1(-56 - 56) - 1(-48 - 48) + (42 - 42) = -16 \neq 0$$

So, A^{-1} exists and the solution of the given system of equations is given by $X = A^{-1}B$

$$\therefore \text{ adj } A = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

MPP-3 CLASS XII ANSWER KEY

1. (d) 2. (b) 3. (a) 4. (d) 5. (d)
 6. (b) 7. (a, c) 8. (b, c, d) 9. (d) 10. (a, c, d)
 11. (a, b, c) 12. (c) 13. (a, c, d)
 14. (d) 15. (d) 16. (a) 17. (1) 18. (1)
 19. (2) 20. (4)

$$\text{So, } A^{-1} = \frac{1}{|A|} (\text{adj } A) = -\frac{1}{16} \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Hence, the solution is given by

$$X = A^{-1}B = -\frac{1}{16} \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{16} \begin{bmatrix} -16000 \\ -35200 \\ -28800 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

$\Rightarrow x = 1000, y = 2200$ and $z = 1800$

Hence, three investments are of ₹ 1000, ₹ 2200 and ₹ 1800 respectively.

19. We know that $A(\text{adj } A) = |A|I$

$$\Rightarrow A = |A|I(\text{adj } A)^{-1} \Rightarrow A = 2(\text{adj } A)^{-1} [\because |A| = 2]$$

$$\Rightarrow A = 2B^{-1}, \text{ where } B = \text{adj } A$$

$$\text{Now, } B = \text{adj } A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore |B| = |\text{adj } A| = |A|^{3-1} = |A|^2 = 4 \quad [\because |\text{adj } A| = |A|^{n-1}]$$

Let C_{ij} be the cofactor of b_{ij} in matrix B . Then,

$$C_{11} = 4, C_{12} = -2, C_{13} = 2, C_{21} = -2, C_{22} = 2, C_{23} = -2, C_{31} = 2, C_{32} = -2 \text{ and } C_{33} = 6$$

$$\therefore \text{adj } B = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 6 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 6 \end{bmatrix}$$

$$\text{and } B^{-1} = \frac{1}{|B|} (\text{adj } B) = \frac{1}{4} \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 6 \end{bmatrix}$$

$$\text{Hence, } A = 2(B)^{-1} = \frac{1}{2} \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

20. We have, $f(x) = x^2 - 4x + 7$

$$\therefore f(A) = A^2 - 4A + 7I_2$$

$$\text{Here, } A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$\text{Now, } A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} \text{ and } 7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\therefore f(A) = A^2 - 4A + 7I_2$$

$$\Rightarrow f(A) = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

Now, $f(A) = O$

$$\Rightarrow A^2 - 4A + 7I_2 = O \Rightarrow A^2 = 4A - 7I_2$$

$$\Rightarrow A^3 = 4A^2 - 7I_2A \Rightarrow A^3 = 4(4A - 7I_2) - 7A$$

$$\Rightarrow A^3 = 9A - 28I_2 \Rightarrow A^4 = 9A^2 - 28I_2A = 9(4A - 7I_2) - 28A$$

$$\Rightarrow A^4 = 36A - 63I_2 - 28A = 8A - 63I_2$$

$$\Rightarrow A^5 = 8A^2 - 63I_2A \Rightarrow A^5 = 8(4A - 7I_2) - 63A = -31A - 56I_2$$

$$\Rightarrow A^5 = -31 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -62 & -93 \\ 31 & -62 \end{bmatrix} + \begin{bmatrix} -56 & 0 \\ 0 & -56 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

TELLING NUMBERS

In 4 years, MCA seats down by over a third, engg seats by a fifth

THE ALL India Council for Technical Education has reduced the intake for its courses by 8%, from 35.52 lakh last year to 32.62 lakh this year. Technical education includes UG, PG and diploma courses in engineering, management, pharmacy, architecture, MCA, hotel management, and applied arts & crafts. Only pharmacy seats have increased this year, marginally. In the last four years, intake across technical courses has fallen by 18% (from 39.61 lakh in 2014-15 to the current 32.62 lakh), with the largest slash coming in Master of Computer Applications.

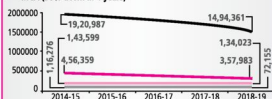
All technical seats (18% down in 4 years)

TOTAL SEATS

2014-15	39,61,662
2015-16	38,35,795
2016-17	37,01,674
2017-18	35,52,199
2018-19	32,62,262

Course by course

- Engineering (22% down in 4 years)
- Management (21% down in 4 years)
- Pharmacy (7% down in 4 years)
- MCA (38% down in 4 years)



MPP-3 MONTHLY Practice Problems

Class XII



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Matrices and Determinants

Total Marks : 80

Time Taken : 60 Min.

Only One Option Correct Type

- If $D = \text{diag}(d_1, d_2, d_3, \dots, d_n)$ where $d_i \geq 0 \forall i$, then D^{-1} equals
 - D
 - $\text{diag}(d_1^n, d_2^n, \dots, d_n^n)$
 - I_n
 - $\text{diag}(d_1^{-1}, d_2^{-1}, d_3^{-1}, \dots, d_n^{-1})$
- Which of the following statements is correct?
 - A skew symmetric matrix of odd order is always invertible
 - A skew symmetric matrix of odd order is always non-invertible.
 - A skew symmetric matrix of even order is always non-invertible.
 - None of these
- If a, b, c are in G.P. or $x - \alpha$ is a factor of $ax^2 + 2bx + c = 0$, then value of the

$$\text{determinant} \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} \text{ is}$$

- 0
 - 1
 - 2
 - 3
- If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ is equal to
 - A^4
 - A^3
 - $I + A$
 - None of these

- If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 3 & -1 & 9 \end{bmatrix}$, then the value of

$\det(\text{adj}(\text{adj } A))$ equals

- 11
- 121
- 1331
- 14641

- The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$, if
 - x, y, z are in A.P.
 - x, y, z are in G.P.
 - x, y, z are in H.P.
 - xy, yz, zx are in A.P.

One or More Than One Option(s) Correct Type

- A and B are square matrices with the same order. If $AB = BA$ and A and B are skew hermitian, then
 - $A + B$ is skew hermitian
 - AB is skew hermitian
 - BA is hermitian
 - All are true
- Let $a, \lambda, \mu \in \mathbb{R}$. Consider the system of linear equations $ax + 2y = \lambda$, $3x - 2y = \mu$. Which of the following statement(s) is(are) correct?
 - If $a = -3$, then the system has infinitely many solutions for all values of λ and μ .
 - If $a \neq -3$, then the system has a unique solution for all values of λ and μ .
 - If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.
 - If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$.

- Value of $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ equals

- 2
- 1
- 4
- 0

10. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ be a matrix. If $A^{10} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

- (a) number of divisors of a is 11
 (b) a is an odd integer
 (c) $(a + b + d)$ is an even integer
 (d) $a + d$ is a multiple of 13

11. If $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then

- (a) $a = \pm \frac{1}{\sqrt{2}}$ (b) $b = \pm \frac{1}{\sqrt{12}}$
 (c) $c = \pm \frac{1}{\sqrt{3}}$ (d) None of these

12. The value of θ for which the system of linear equations in x, y, z given as
 $(\sin 3\theta)x - y + z = 0$
 $(\cos 2\theta)x + 4y + 3z = 0$
 $2x + 7y + 7z = 0$
 has a non-trivial solution, is/are

- (a) $\frac{n\pi}{2}$ (b) $\frac{2n\pi}{3}$
 (c) $n\pi + (-1)^n \frac{\pi}{6}$ (d) None of these

13. If A is symmetric and B is skew symmetric matrix, then which of the following is/are not true?
 (a) ABA^T is symmetric matrix
 (b) $AB^T + BA^T$ is symmetric matrix
 (c) $(A + B)(A - B)$ is skew symmetric
 (d) $(A + I)(B - I)$ is skew symmetric

Comprehension Type

Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Also,

let $S_n = \alpha^n + \beta^n$ for $n \geq 1$ and $\Delta = \begin{bmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{bmatrix}$

14. If $\Delta < 0$ then the equation $ax^2 + bx + c = 0$ has
 (a) positive real roots (b) negative real roots
 (c) equal roots (d) imaginary roots
15. If a, b, c are rational and one of the roots of the equation is $1 + \sqrt{2}$, then the value of Δ is
 (a) 8 (b) 12 (c) 30 (d) 32

Matrix Match Type

16. Match the following:

	Column-I	Column-II
P.	If ω is a non real cube root of unity, then a root of the following equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is	1. 0
Q.	If $A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{bmatrix}$, then rank of A is	2. -2
R.	Suppose that system of equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ has non trivial solution. Then $a^2 + b^2 + c^2 + 2abc =$	3. 1
S.	$\begin{vmatrix} a+1 & a+2 & a+4 \\ a+3 & a+5 & a+8 \\ a+7 & a+10 & a+14 \end{vmatrix}$	4. 4

P	Q	R	S
(a) 1	3	3	2
(b) 2	4	3	1
(c) 4	2	3	1
(d) 4	1	2	3

Integer Answer Type

17. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $\det(A^{2005})$ equals to
18. If $A^2 = 3A - 2I$ and $A^8 = pA + qI$, then $p + q$ is
19. If A, B and C are $n \times n$ matrices and $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, then the value of $[\det(A^2BC^{-1})]$ (where $[-]$ represents greatest integer function) is
20. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, λ must be

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted
 No. of questions correct
 Marks scored in percentage

Check your score! If your score is

- > 90% EXCELLENT WORK ! You are well prepared to take the challenge of final exam.
 90-75% GOOD WORK ! You can score good in the final exam.
 74-60% SATISFACTORY ! You need to score more next time.
 < 60% NOT SATISFACTORY! Revise thoroughly and strengthen your concepts.

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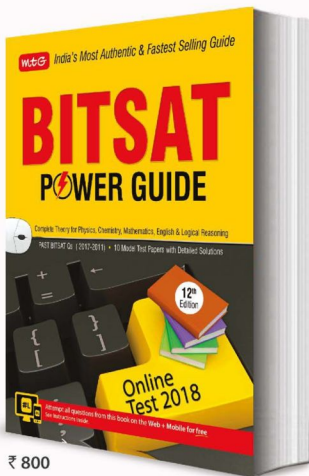
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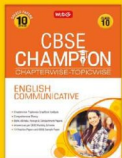
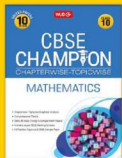
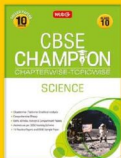
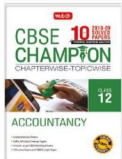
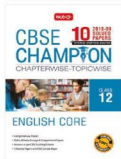
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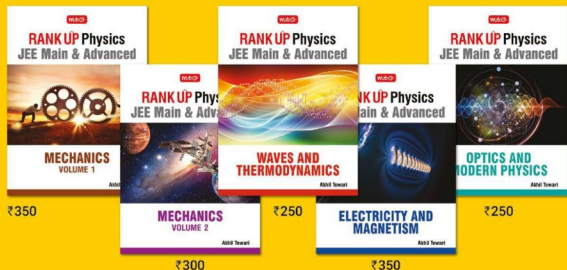


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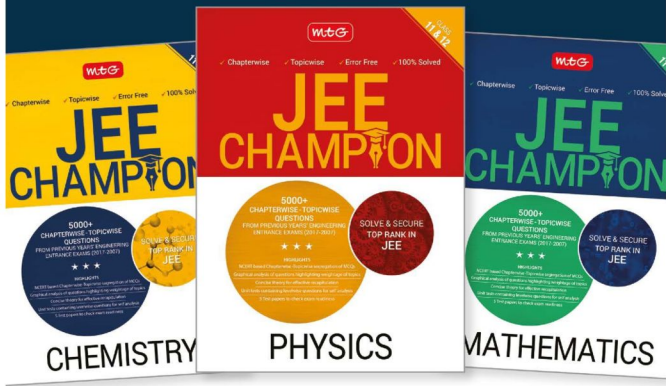
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